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Combined stability of unstiffened cones – Theory, experiments and design codes

O. Ifayefunmi, J. Błachut*

The University of Liverpool, Mechanical Enginering, Brownlow Hill, Liverpool L69 3GH, United Kingdom

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The elastic-plastic buckling of short and relatively thick unstiffened truncated conical shells subjected to axial compression and external pressure is investigated. This is done using numerical and experimental approach. For the numerical analysis, the finite element code is employed to obtain the domain of combined stability. To validate numerical predictions, thirteen nominally identical laboratory scale cones with 26.56° semi-vertex angle and 3 mm nominal wall thickness with integral top and bottom flanges were CNC machined from 252 mm diameter mild steel billet. Two of the models were subjected to axial compression, with further two subjected to pure lateral external pressure, while the remaining nine cones were subjected to combined action of axial compression and external pressure of different ratio. Experimental results compare well with numerical predictions except for pure axial compression. However, the accuracy of these results is strongly dependent on the approach to modeling of material. Experimental results were compared with predictions of failure loads obtained from ASME code case 2286–2, and with the ECCS design rules for the case of axial compression and lateral pressure.

1. Introduction

Cones are important structural components primarily used in the marine and offshore industries. They can buckle in the elastic-plastic range. Typical application of thick cones includes: transition elements between two cylindrical shells of different diameter, piles for holding jackets when driven into the sea bed, and the legs of off-shore drilling rigs. When used as transition components, they are mostly subjected to external pressure. However, when used as piles for jackets holding they are subjected to axial compression. In the case of off-shore drilling rigs, they are under combined loading, i.e., some part of the structures is subjected to external pressure, in addition to axial compressive force in the legs of the drilling rig. Other application includes chemical industry, e.g., as flue gas desulphurization (FGD) vessel assembly.

Review of known tests on conical shells between 1958 and 2008 under various loading conditions is reported in Refs [1–4]. Refs [1–3] show the number of tests per year, the type of material from which shells were made and the type of load applied together with the source of data. Ref. [1] is devoted to unstiffened cones, while Refs [2,3], cover experiments on stiffened cones and detail the type of stiffeners used in each test. Also, Ref. [4] provides details about material properties of tested cones which are discussed in the current paper.

The majority of buckling tests on unstiffened cones were carried out under the application of single loading, i.e., axial compression, external pressure or torsion, etc. Details about elastic buckling tests of conical shells subjected to simultaneous action of two or more loads can be found in Refs [5–14]. It appears that there have been no experiments within the elastic-plastic range for unstiffened cones subjected to combined axial compression and external pressure, except for the partial results being reported in Refs [2,3]. Results of experimental and numerical study into buckling strength of ring-stiffened steel cones are reported in Ref. [15]. Two competing forms of ring reinforcement are discussed: (i) cones with sparsely distributed but heavy rings, and (ii) cones with densely distributed rings of smaller stiffness. In the first case cones tend to fail by the inter-ring skin. In the second case cones failed through the global instability. In both cases the load applied was quasi-static hydrostatic pressure. References are also made to the available design procedures for metallic cones.

Buckling strength of conical shells is not immune from the effects of initial geometric imperfections. There is a wide body of research results for elastic buckling and only few papers address elastic-plastic cases, Ref. [16]. The effect of inward, dimple-type axisymmetric imperfection on elastic buckling of axially compressed cones has been studied in Ref. [17]. This experimental study confirmed the fact that the inward dimple of small amplitude

* Corresponding author. Tel.: +44 151 7944817.
E-mail address: em20@liverpool.ac.uk (J. Błachut).

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can significantly lower the magnitude of buckling load. A recent study, Ref. [18], has examined the effect of outward, dimple-type axisymmetric imperfection on elastic-plastic buckling of axially compressed cones. It is reported that the outward dimple is equally dangerous shape imperfection as the inward one. For some profiles, the outward axisymmetric imperfection can be more dangerous than the inward one.

The present paper contains the first test data along the entire combined stability envelope for relatively thick mild steel cones subjected to axial compression and external pressure acting simultaneously. As mentioned above, partial results have been published in Refs [1–3] but for completeness these results are also included in this paper in addition to the full set of experimental data. Experimental results are compared with the corresponding FE predictions and known design rules.

2. Background

Consider a truncated conical shell with small and big radii, r_1, and r_2, respectively, uniform wall thickness, t, height of the shell, h, and the cone angle, β. Assume that the cone can be subjected to axial compression and external pressure acting simultaneously, as shown in Fig. 1. Assume that cones are clamped at the larger radius end. They are allowed to move axially at the smaller radius end – as illustrated in Fig. 1. Assume that the material is modeled as elastic-perfectly plastic with Young's Modulus, $E = 210.49$ GPa, yield stress, $\sigma_{yp} = 230.6$ MPa and Poisson's ratio, $\upsilon = 0.281$ (further details about material properties can be found in Refs [1,4]). A parametric study was carried out for the above data in order to investigate the influence of the radius-to-wall-thickness ratio, (r_2/t) on: (i) the shape of the combined stability domain, and (ii) the yield envelope. Results were obtained using ABAQUS FE code [19] for cones with geometries characterized by $34.3 \leq r_2/t \leq 750$ (with $r_2/r_1 = 2.02$, $h/r_2 = 1.01$, and $\beta = 26.56^\circ$). The FE analyses were both axisymmetric and two dimensional. During FE calculations, two types of analysis have been performed, viz: (i) Bifurcation buckling analyses, and (ii) Collapse analyses. The first yield loads of the models were established using the post-processing procedure discussed in Ref. [4]. A number of steps were required to obtain the interactive curve. They are listed below for the case of collapse:

(i) calculate the collapse force under pure axial compression
(ii) calculate the collapse pressure under pure lateral pressure
(iii) calculate the collapse load under combined loading, i.e., axial compression and external pressure acting simultaneously using a combination of loading paths
(iv) calculate the first yield load for cone under axial compression, external pressure and under combined loading.

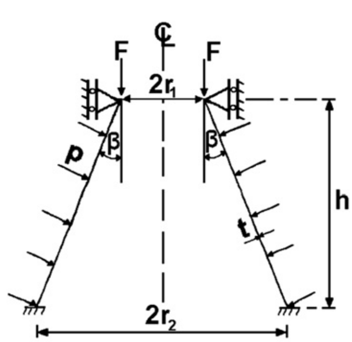

Fig. 1. Cone subjected to axial compression and external pressure acting simultaneously.

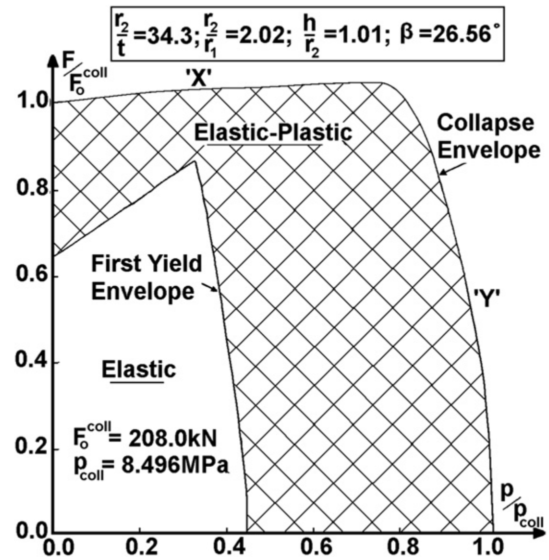

Fig. 2. Combined stability plot.

Results for (i), (ii) and (iii) were extracted using RIKS method. Results of (iv) were obtained by post-processing the output. Fig. 2 shows a typical first yield and collapse envelopes obtained from tasks (i)–(iv).

2.1. Finite element convergence studies

The number of axial and circumferential elements to be used was studied first. The convergence study was carried out for static analysis using Riks method. SAX2 and S8R FE models were part of this investigation. Table 1 gives collapse pressures, at constant axial force of 100 kN, for variable number of axial elements (SAX2 model). Similarly, in Table 2 the collapse pressures, at constant axial force of 100 kN, are given for varying number of eight-node S8R shell elements in either the axial or hoop directions.

It can be seen from Table 2, that the number of elements in the hoop direction does not affect the collapse pressure. Therefore, it was decided to use the 70-element model for SAX2 case, and 142 (axial) × 40 (hoop) elements for S8R model. In summary, the SAX2 models had a total of 70 elements and 140 nodes whereas, the S8R model had 5680 elements and 17,120 nodes.

2.2. Combined loading – external pressure and axial compression

Buckling behavior of cones subjected to combined action of external pressure, p, and an independent axial compression, F, acting simultaneously depends on the relative magnitude of, p, and, F. It is customary to represent the buckling strength of such cones through the so called combined stability plots (interactive diagrams).

Table 1
Convergence study for a cone under combined loading using SAX2 shell elements.

Number of elements	10	30	40	50	60	70	80
Constant Axial Force (kN)	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Collapse pressure (MPa)	8.286	8.212	8.205	8.203	8.201	8.200	8.201