International Journal of Pressure Vessels and Piping 89 (2012) 19–27

Contents lists available at SciVerse ScienceDirect

International Journal of Pressure Vessels and Piping

journal homepage: www.elsevier.com/locate/ijpvp

Comparison of existing plastic collapse load solutions with experimental data for 90° elbows

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ARTICLE INFO

Article history:
Received 11 July 2011
Accepted 14 September 2011

Keywords:
Finite element analysis
Elbow
Internal pressure
In-Plane bending
Limit load

ABSTRACT

This paper compares published experimental plastic collapse loads for 90° elbows with existing closed-form solutions. A total of 46 experimental data are considered, covering pure bending (in-plane closing, in-plane opening and out-of-plane bending) and combined pressure and bending loads. The plastic collapse load solutions considered are from the ASME code, the Ductile Fracture handbook of Zahoor, by Chattopadhyay and co-workers, and by Y.-J. Kim and co-workers. Comparisons with the experimental data shows that the ASME code solution is conservative by a factor of 2 on collapse load for in-plane closing bending, 2.3 for out-of-plane bending, and 3 for in-plane opening bending. The solutions given by Kim and co-workers give the least conservative estimates of plastic collapse loads, although they provide slightly non-conservative estimates for some data.

1. Introduction

There have been numerous published papers in the literature on plastic limit and collapse analysis of elbows [1–31]. Although analytical [1–3] and experimental studies [4–11] are extremely useful, it is difficult to perform systematic analyses to include all relevant variables affecting plastic behavior and plastic limit/collapse loads. Complexities in finding plastic limit/collapse loads for elbows may result from the following sources; for example, the first one is the geometry. Compared to a straight pipe, an elbow has two more dimensions, namely the bend radius and bend angle. Furthermore, in practice, an elbow is always attached to straight pipes. The role of the attached straight pipe in plastic behavior of an elbow could be significant, due to possible stress redistribution to the attached pipe after plastic yielding [24]. As an elbow is a flexible component, the large geometry change effect on collapse behavior can also be significant. Consequently, the loading mode and in-plane closing or opening bending, affect plastic collapse loads due to the cross-sectional shape change during bending. In-plane closing bending leads to a geometric weakening effect and plastic collapse loads can be lower than limit loads obtained using geometrically linear analysis. In-plane opening bending, on the other hand, shows a geometric strengthening effect. A further problem associated with the large geometry change effect is the definition of the plastic collapse load. When the small strain (or geometrically linear) option is used together with an ideal elastic-perfectly plastic material, the resulting load–displacement curve shows a clear limiting load and thus a limit load can be easily defined. However, when the non-linear geometry option is chosen, the resulting load–displacement curve does not show a clear limiting load and can change continuously. Accordingly a plastic collapse load can be defined in several ways. One popular way to define a plastic collapse load is to use the twice-elastic-slope (TES) line [32]. However, the consequence of such a definition is the effect of material properties on the calculated plastic collapse load. As the initial bending stiffness of an elbow depends on material properties, the plastic collapse load defined by the intersection with the TES line can depend on those properties [28].

Considering the complexities discussed above, limit analysis using finite element (FE) analysis is quite attractive. For instance, extensive FE results for elbows have been reported recently, leading to some closed-form approximations to the plastic collapse and limit loads for elbows [21–31]. However, the existing proposed solutions often give different estimates, and thus it would be useful to compare these solutions with experimental data.

This paper compares published experimental plastic collapse loads for 90° un-cracked elbows with existing closed-form analytical solutions. The experimental data include those for in-plane closing, in-plane opening and out-of-plane bending and for combined pressure and bending. Section 2 presents four existing

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0308-0161/$ – see front matter
doi:10.1016/j.ijpvp.2011.09.001

Nomenclature

B_1, B_2	B-stress indices, see Eq. (3a)
E	Young's modulus
I	second moment of inertia
M	bending moment
M_o	plastic collapse moment of an elbow for an arbitrary ε_o value
M_o^{exp}	experimentally measured plastic collapse moment of an elbow
M_o^{ref}	plastic collapse moment of an elbow for $\varepsilon_o = 0.001$
M_o^A	plastic collapse moment solution of an elbow from ASME BPVC
M_o^D	collapse moment solution of an elbow from Ductile Fracture Handbook.
M_o^C	collapse moment solution of an elbow by Chattopadhyay and co-workers.
M_o^K	collapse moment solution of an elbow by Kim and co-workers.
P, p	internal pressure and normalized pressure, see Eq. (7b)
R	bend radius
r	mean pipe radius
t	thickness of pipe
ε_o	yield strength-to-elastic modulus ratio, $=\sigma_o/E$
λ	bend characteristic, $=Rt/r^2$
σ_o	limiting strength of an elastic-perfectly plastic material; 0.2% proof (yield) strength for a strain hardening material
ASME BPVC	American Society Mechanical Engineers Boiler and Pressure Vessel Code
TES	twice-elastic-slope

solutions for the plastic collapse loads of 90° un-cracked elbows. Experimental data are briefly summarized and compared with the existing analytical solutions in Section 3. Section 4 concludes the paper.

2. Existing plastic collapse load solutions: review

Fig. 1 depicts a 90° elbow, considered in the present work. The mean radius and thickness of the pipe are denoted by r and t, respectively, and the bend radius by R.

2.1. Solution in the ASME BPVC Sec III [32]

In Section III of the ASME (American Society Mechanical Engineers) BPVC (Boiler and Pressure Vessel Code) [32], the primary stress limit for piping components is given by

$$B_1\frac{P(r+t/2)}{t}+B_2\frac{(r+t/2)}{I}M \le 1.5S_m \tag{1a}$$

where B_1 and B_2 are the B-stress indices; P and M are internal pressure and bending moment, respectively; t is the nominal wall thickness; S_m is the maximum allowable stress intensity value; and I is the second moment of inertia:

$$I = \frac{\pi}{4}\left[\left(r+\frac{t}{2}\right)^4-\left(r-\frac{t}{2}\right)^4\right] \tag{1b}$$

This equation is applicable to both straight pipes and elbows and reduces to $I=\pi r^3 t$ for the thin-walled cylinder where $t/r \ll 1$. For

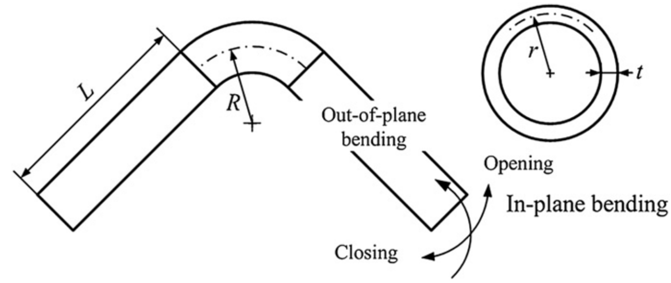

Fig. 1. Schematic diagram for 90° elbows.

straight pipes, for instance, the B-stress indices are given as $B_1 = 0.5$ and $B_2 = 1.0$. Assuming $S_m = \sigma_o$ (where σ_o denotes the yield strength of the material), for thin-walled pipes under pure bending ($P = 0$), Eq. (1) predicts the limit moment M_o as

$$M_o = 1.5\pi r^2 t\sigma_o \tag{2}$$

This is about 18% higher than the exact limit moment, $M_o = 4\sigma_o r^2 t$. For elbows, the B-stress indices are given by

$$B_1 = \begin{cases} 0 \text{ for } -0.1+0.4\lambda \le 0 \\ -0.1+0.4\lambda \text{ for } 0 < -0.1+0.4\lambda < 0.5 \\ 0.5 \text{ for } 0.5 \le -0.1+0.4\lambda \end{cases}$$
$$B_2 = \begin{cases} 1 \text{ for } 1.30/\lambda^{(2/3)} \le 1.0 \\ 1.30/\lambda^{(2/3)} \text{ for } 1.0 < 1.30/\lambda^{(2/3)} \end{cases} \tag{3a}$$

where the bend characteristic λ is given by

$$\lambda = \frac{Rt}{r^2} = \frac{(R/r)}{(r/t)} \tag{3b}$$

There is no restriction on the loading mode, and it is believed that the above equation is based on the worst case loading mode. For pure bending, Eq. (1) with $B_2 = 1.30/\lambda^{(2/3)}$ assuming $S_m = \sigma_o$ gives

$$M_o^A = 0.906\lambda^{2/3}\sigma_o(2r)^2 t \tag{4}$$

This can be compared with the existing analytical solutions of Calladine [2]:

$$M_o = 0.935\lambda^{2/3}\sigma_o(2r)^2 t \tag{5}$$

2.2. Ductile fracture handbook solution [33]

In the ductile fracture handbook [33], the following limit load solution is given for elbows subjected to in-plane bending moment.

$$M_o^D = 0.935(2r)^2 t\sigma_f\lambda^{2/3} \tag{6}$$

where σ_f denotes the flow strength, usually defined as the average of yield and ultimate strengths. The geometric restriction of Eq. (6) is given by $2 \le R/r \le 3$, $\lambda < 0.5$ and $r/t \le 7.5$. Furthermore, it is valid only for in-plane closing and opening bending, not for out-of-plane bending. It should be noted that Eq. (6) follows simply from the analytical solution of Calladine, Eq. (5), by replacing the yield strength by the flow strength.

2.3. Solutions by Chattopadhyay and co-workers

A series of papers have been published by Chattopadhyay and co-workers on plastic collapse load solutions for elbows, based on