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Analytical solution for stress, strain and plastic instability of pressurized pipes with volumetric flaws

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ABSTRACT

The mechanical behavior of internally pressurized pipes with volumetric flaws is analyzed. The two possible modes of circumferentially straining the pipe wall are identified and associated to hypothesized geometries. The radial deformation that takes place by bending the pipe wall is studied by means of axisymmetric flaws and the membrane strain developed by unequal hoop deformation is analyzed with the help of narrow axial flaws. Linear elastic shell solutions for stress and strain are developed, the plastic behavior is studied and the maximum hoop stress at the flaw is related to the undamaged pipe hoop stress by means of stress concentration factors. The stress concentration factors are employed to obtain equations predicting the pressure at which the pipe fails by plastic instability for both types of flaw. These analytical solutions are validated by comparison with burst tests on 3″ diameter pipes and finite element simulations. Forty-one burst tests were carried out and two materials with very dissimilar plastic behavior, carbon steel and austenitic stainless steel, were used in the experiments. Both the analytical and the numerical predictions showed good correlation with the experimentally observed burst pressures.

1. Introduction

Pipeline failure statistics [1,2,3,4,5] rank corrosion between the 1st and the 3rd leading cause of pipeline failures and indicate that this failure mode is responsible for 15–27% of the total of recorded failures. In fact, the loss of material in the wall of a pipeline by action of a corrosive process is a very common situation and the safe operation of hazardous liquid and gas pipelines does demand a judicious methodology to evaluate volumetric flaws.

The need of a criterion to assess corrosion flaws was recognized by Kiefner et al. [6], who proposed the arguably first methodology for evaluation of the burst pressure of a steel pipeline with a volumetric flaw:

$$\frac{\sigma_\theta}{\overset{*}{\sigma}} = \frac{1-\dfrac{d}{t_2}}{1-\left(\dfrac{d}{t_2}\right)F^{-1}},\ \sigma_\theta = \frac{pR}{t_2},\ \overset{*}{\sigma} = Sy + 10ksi,$$

$$F = \sqrt{1+1.255\frac{L^2}{Rt_2} - 0.0135\frac{L^4}{R^2t_2^2}} \qquad (1)$$

Eq. (1) was proposed after an experimental research in which 48 pipeline sections with machined surface grooves were burst. The expression for the bulging factor was the one derived by Erdogan and Kibler [7], refining a research by Folias [8], for the case of a through wall axial crack.

In 1984 the American Society of Mechanical Engineers introduced the first standard for evaluation of corroded steel pipelines, the ASME B31G. Besides Kiefner experiments, several hundred rupture tests of field corroded pipes were employed to formulate this methodology [9].

Kiefner and Veith [10], following an experimental research comprising a total of 86 burst tests, modified Eq. (1), replacing the ratio between the flaw depth to the pipe wall (d/t_2) by the ratio between the removed and the original area. For defects of constant depth, the two equations are equivalent. Afterward, Kiefner augmented his pipe failures database to 124 results [11] and finally performed additional 129 burst tests [12].

Another standard for evaluation of corroded steel pipelines was introduced in 1999 by the Det Norske Veritas, the DNV RP-F101 [13]. It was calibrated by more than 400 finite element method (FEM) analyses [14]. The FEM simulations themselves were validated by 12 burst tests carried out by DNV and 70 burst tests previously performed by BG Technology [15].

The equations proposed by Kiefner remain in use in almost all standards or procedures for evaluation of corroded pipelines up to

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Nomenclature

E	Young's Modulus
E'	$E/(1-\nu^2)$
F	bulging or Folias factor
K	plastic stiffness modulus
L	defect half-length
M	bending moment per unit length
N	normal force per unit length
Q	shearing force per unit length
R	pipe radius
Su	tensile test ultimate stress
Sy	tensile test yield stress
T	$(t_2/t_1)^{0.5}$
d	defect depth
e	Euler's constant (2.718…)
f	stress concentration factor
n	strain hardening exponent
p	pressure
t	thickness
α	non dimensional half-length of the defect
β, β_E, β_m	length non-dimensionalization parameters
β_{ND}	component of β_m
ε	strain
$\bar{\varepsilon}$	equivalent strain
ν	Poisson's ratio
κ	width of influence of a defect (rad)
σ	normal stress
$\bar{\sigma}$	von Mises' equivalent stress
$\overset{*}{\sigma}$	critical or failure stress
τ	shear stress
l	defect half-width in radians
r, θ, z	cylindrical coordinates
u, v, w	deformations in cylindrical coordinates (associated to r, θ and z respectively)

Subscripts

1,2	in the defect; out of the defect.
o	initial dimension
E	elastic

this date. They are the basis for the revised version of the ASME B31G [16] and the DNV RP-F101 [13]. Comparing the different procedures, one finds changes in the value of the critical (or flow) stress, in the expression utilized for the bulging factor F and in the idealized flaw profile.

The mathematical description of plastic instability was published by A. Considère [17] in 1885. Its application to pipes or thin wall cylinder vessels under internal pressure is well established [18]. Recent research discussed which yield model provides the best prediction for the burst pressure of an unflawed pipe section [19,20]. Nevertheless, very few researchers ventured analytical solutions for the plastic instability of a pipeline with a volumetric flaw. Kanninen et al. [21] studied stress and strain in the elastic domain of axisymmetric and infinitely long flaws. Stewart et al. [22] studied the phenomenon of plastic instability of infinitely long flaws.

Despite the remarkable research effort on determining the remaining strength of corroded pipelines, a complete knowledge of the behavior of a pipeline with a volumetric flaw has not been attained yet. This paper presents analytical solutions for stress, strain and plastic instability of a pressurized thin wall cylinder with a volumetric discontinuity in its wall.

Initially, the two possible straining modes that can take place at a volumetric flaw on a pressurized cylinder are identified and each one is associated to a hypothesized geometry. The radial deformation that takes place by bending the pipe wall is studied by means of axisymmetric flaws, the membrane strain generated by non-uniform circumferential deformation is analyzed using narrow axial flaws. Analytical solutions for deformation, strain and stress in the linear elastic domain are developed for both geometries. These solutions are modified to account for the plastic behavior and the maximum hoop stress in the flaw is related to the undamaged pipe hoop stress by stress concentration factors. Finally, making use of the stress concentration factors, the burst pressure is determined by calculating the maximum point of the pressure vs. effective strain curve.

The analytical solutions for the instability pressure are validated by comparison with burst tests on 3″ diameter pipes and finite element method (FEM) simulations.

2. Preliminary concepts

The use of cylindrical coordinates is natural when studying a cylindrical shell. The definition of the cylindrical coordinate system and of the material strains in this coordinate system can be found in different texts, as Timoshenko and Goodier [23] or Sokolnikoff [24]:

$$\varepsilon_r = \frac{\partial u}{\partial r},\ \varepsilon_\theta = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + u\right),\ \varepsilon_z = \frac{\partial w}{\partial z},$$
$$\varepsilon_{rz} = \frac{1}{2}\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right),\ \varepsilon_{\theta z} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial z}\right),\ \varepsilon_{r\theta} = \frac{1}{2}\left[\frac{\partial v}{\partial r} + \frac{1}{r}\left(\frac{\partial u}{\partial \theta} - v\right)\right] \quad (2)$$

The hoop stress is the principal stress of greater magnitude in a pressurized cylinder. Consequently, the hoop strain ε_θ is the main focus of this study.

Eq. (2) shows that the hoop strain has two components: u/r and $1/r(\partial v/\partial\theta)$. In absence of torsion, an axisymmetric defect does not present the $1/r(\partial v/\partial\theta)$ component and, therefore, enables the study of the u/r component by itself. Fig. 1 illustrates this type of flaw and its dimensions.

The narrower the flaw is, the less the pipe wall can bulge in the flaw. Therefore, if the flaw is sufficiently narrow, the variation of the u/r component of the strain in the angular direction can be neglected, what enables the analytical solution of the $1/r(\partial v/\partial\theta)$ component of the strain. Fig. 2 illustrates a narrow axial flaw and its dimensions.

Any structure in which the increase of load causes the reduction of the withstanding area has a maximum load carrying capacity. The maximum load occurs when the rate of increase of strength equals the rate of reduction of the load bearing capacity caused by the area reduction. At this point, the load derivative with respect to the strain is null:

$$\frac{dP}{d\varepsilon} = 0 \Rightarrow \frac{d(\sigma A)}{d\varepsilon} = 0 \therefore \frac{d\sigma}{d\varepsilon}A + \sigma\frac{dA}{d\varepsilon} = 0 \quad (3)$$

In the plastic region, the elastic components of the strains usually can be neglected. If this is the case, Ludwick's equation