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Limit and plastic collapse loads for un-reinforced mitred bends under pressure and bending

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ABSTRACT

Approximate limit and plastic collapse load solutions for un-reinforced mitred bends under internal pressure and under bending are proposed in this paper, based on three-dimensional finite element analysis and approximate solutions for smooth bends. Solutions are given for single- and multi-mitred bends (mainly for single and double segmented bends) with the pipe mean radius-to-thickness ratio (r/t) ranging from $r/t = 5$ to $r/t = 50$, and the bend radius-to-mean radius ratio (R/r) from $R/r = 2$ to $R/r = 4$. Internal pressure, in-plane bending and out-of-plane bending loads are considered, but not their combination. It is found that the essential features of limit and plastic collapse loads for mitred bends are similar to those for smooth bends, and thus existing solutions for smooth elbows can be used to construct limit loads and plastic collapse for mitred bends.

1. Introduction

Due to their economic benefits, mitred bends are often used for large-diameter pipework or ducting in chemical plants. A recent review of the literature for the structural assessment of mitred bends [1] suggests that information on limit[1] and plastic collapse[2] loads for mitred bends is very limited, despite the fact that such information is essential for design and assessment. In the literature, in fact, the authors have found only a few papers related to this issue and they considered only single-mitred bends [2,3]. In contrast, there have been many published papers on limit and plastic collapse loads for smooth bends [4–20]. As smooth bends are limiting cases of mitred bends, information on limit and plastic collapse loads for smooth bends would be useful. In particular, recent works on limit and plastic collapse loads for smooth bends, published by the authors and co-workers [15–20], are worth noting. Based on three-dimensional (3-D) finite element (FE) limit analysis using elastic-perfectly plastic materials, the authors proposed closed-form approximations for limit pressures and plastic collapse bending moments (defined by the twice-elastic-slope method [21]) of smooth bends. The solutions were based on FE results covering mean radius-to-thickness ratios from two to fifty with various values of the bend radius-to-pipe radius ratio. For loading, all possible loading conditions were considered; internal pressure, in-plane closing bending, in-plane opening bending and out-of-plane bending.

This paper proposes approximate limit and plastic collapse loads for un-reinforced mitred bends, based on 3-D FE limit analysis using elastic-perfectly plastic materials. Noting that smooth bends are limiting cases of mitred bends, limit and plastic collapse load solutions for smooth bends are taken as the reference solutions. By comparing FE results with these reference solutions, approximate limit and plastic collapse load solutions are proposed for single- and multi-mitred bends under internal pressure, in-plane bending and out-of-plane bending. Section 2 describes the FE limit analysis performed in this work. Limit and plastic collapse load solutions for smooth bends are summarised in Section 3. Plastic collapse load solutions for mitred bends are presented for bending in Section 4 and for internal pressure in Section 5. Conclusions are given in Section 6.

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[1] The limit load is obtained using FE analysis by incorporating an elastic-perfectly-plastic material model and small displacement theory.

[2] The plastic collapse load can be obtained using FE analysis by incorporating an elastic–plastic material behavior with the non-linear geometry effect. In the present work, it is obtained by incorporating an elastic-perfectly-plastic material model with the non-linear geometry effect. Definition of the collapse load, adopted in this paper, will be explained in Section 2.3.

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Nomenclature

a	segment length in a multi-mitred bend, Fig. 1b
E	Young's modulus
L	length of the straight pipe attached to a mitred bend
n	number of segments
M, P	bending moment and internal pressure, respectively
M_o, P_o	plastic collapse moment and limit pressure of a bend, respectively
M_o^s, P_o^s	limit moment and pressure of a straight pipe, see Eq. (3c) and Eq. (6b), respectively
M_o^{ref}	plastic collapse moment of a bend for $\varepsilon_o = 0.001$
R	bend radius
r	mean pipe radius
t	pipe thickness
λ	bend characteristic, $= Rt/r^2$
θ	half angle of the segment
Φ	bend angle
ν	Possion's ratio
σ_o	limit strength of an elastic-perfectly-plastic material
ε_o	$=\sigma_o/E$

Abbreviations

est	estimated value using closed-form approximations
FE	finite element
Seg	number of segments
TES	twice-elastic-slope
3-D	three-dimensional

2. Finite element limit analysis

2.1. Geometry and loading

Fig. 1 schematically depicts un-reinforced single- and multi-mitred bends (with single and double segments), together with a smooth bend. The mean radius and thickness of the pipe are denoted by r and t, respectively. The bend angle is denoted by ϕ. In the present work, the values of r/t were ranged from $r/t = 5$ to $r/t = 50$, and those of ϕ from 0 to 90° ($=\pi/2$). Note that $\phi = 0$ corresponds to the straight pipe. For smooth bends, the non-dimensional bend characteristic, λ, is defined by

$$\lambda = \frac{Rt}{r^2} = \frac{(R/r)}{(r/t)} \tag{1}$$

where R denotes the bend radius. It is known that the bend characteristic is an important parameter for limit and plastic collapse loads of bends [4,5]. For single-mitred bends (Fig. 1a), the bend radius cannot be defined. For multi-mitred bends, on the other hand, the bend radius R can be defined, as schematically shown in Fig. 1b. For instance, the bend radius R and the segment length in the neutral axis, a, are related to

$$\theta = \frac{\pi}{4(1+n)}; \quad 2R\tan\left[\frac{\pi}{4(1+n)}\right] = a \tag{2}$$

where θ is the half angle for the segment (Fig. 1b), and n denotes the number of segments in the multi-mitred bend; e.g. $n = 1$ in Fig. 1b and $n = 2$ in Fig. 1c. The smooth bend then corresponds to the limiting case $n \to \infty$,

The piping system considered comprises a bend attached to sufficiently long straight pipes, as depicted in Fig. 1. For smooth bends, it was found that the effect of the length of the attached straight pipe, L, on plastic behavior is minimal, as long as it is longer than four times the pipe radius, $L = 4r$ [16]. The length of the attached straight pipe in this work was taken to be sufficiently long, twenty times the pipe radius, $L = 20r$, for all cases.

2.2. Finite element analysis

Fig. 2 depicts a typical FE mesh. To reduce computing time, reduced integration elements (element type C3D20R within

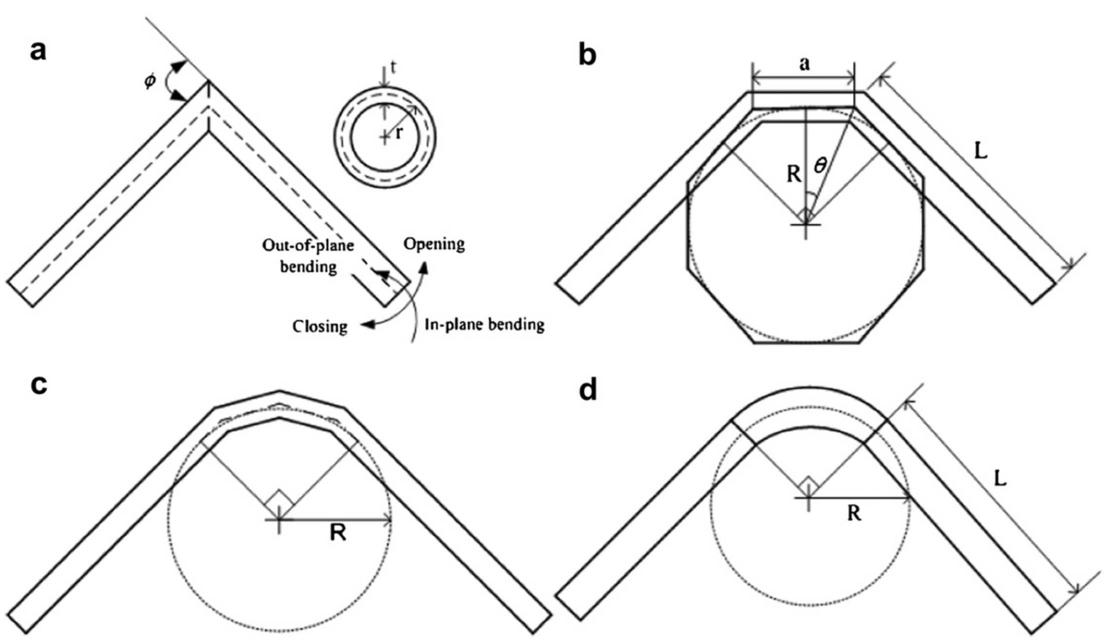

Fig. 1. Schematic figures for bends: (a) single-mitred bend, (b) multi-mitred bend with single segment, (c) multi-mitred bend with double segments, and (d) smooth bend.