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An anisotropic tertiary creep damage constitutive model for anisotropic materials

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ABSTRACT

When an anisotropic material is subject to creep conditions and a complex state of stress, an anisotropic creep damage behavior is observed. Previous research has focused on the anisotropic creep damage behavior of isotropic materials but few constitutive models have been developed for anisotropic creeping solids. This paper describes the development of a new anisotropic tertiary creep damage constitutive model for anisotropic materials. An advanced tensorial damage formulation is implemented which includes both material orientation relative to loading and the degree of creep damage anisotropy in the model. A variation of the Norton-power law for secondary creep is implemented which includes the Hill's anisotropic analogy. Experiments are conducted on the directionally-solidified bucket material DS GTD-111. The constitutive model is implemented in a user programmable feature (UPF) in ANSYS FEA software. The ability of the constitutive model to regress to the Kachanov-Rabotnov isotropic tertiary creep damage model is demonstrated through comparison with uniaxial experiments. A parametric study of both material orientation and stress rotation are conducted. Results indicate that creep deformation is modeled accurately; however an improved damage evolution law may be necessary.

1. Introduction

In the power generation and aerospace industries, components such as pressure vessels, pipes, gas turbine disks and vanes, and turbine blades experience high temperatures such that creep deformation will occur. In the case of industrial gas turbines where the cycle duration and maintenance intervals can be in the thousands of hours, and there are drives to increases temperature and pressure, careful selection and accurate prediction of material behavior is paramount; therefore, directionally-solidified (DS) materials have been implemented to minimize intergranular (brittle) creep cracking by alignment of long grains with the first principal stress direction [1]. Typically DS gas turbine blade materials are transversely-isotropic, consisting of a columnar microstructure where there is a plane of "transverse grain (T)" isotropy and an enhanced "longitudinal grain (L)" orientation. Creep and stress-rupture properties are one of the more important variables in the overall life of turbine blades [2].

In the case of welded pressure vessels, welding is a directional solidification process. A single weld bead consists of a single columnar solidification microstructure. Multi-pass welding (used on pressure vessels) will produce a transversely-isotropic microstructure [3].

In the case of thin-walled pipes used in power plants, considerable work has been focused around isotropic creep damage modeling [4,5]. Literature has demonstrated that strength anisotropy in thin-walled tubular elements is common [6].

Accurate, modeling of the creep deformation and damage behavior of transversely-isotropic materials is important. A novel anisotropic creep damage model for transversely-isotropic materials is developed based on the Kachanov-Rabotnov isotropic formulation [7,8]. Experiments are conducted on uniaxial specimen of the bucket material DS GTD-111. The constitutive model is implemented in Finite Element Analysis (FEA) software. A comparison between the experiments, Kachanov-Rabotnov model, and novel constitutive model is conducted. An examination of the strain tensor is provided. A parametric exercise of the constitutive model for various material orientations and states of stress demonstrates functionality.

2. Continuum damage mechanics

A damage mechanism is a manifestation of the degradation of the microstructure of a material and can occur in two forms: transgranular (ductile) damage and intergranular (brittle) damage. Transgranular (ductile) damage arises where slip bands of plasticity

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form under high stress and low temperature. Intergranular (brittle) damage is a microcracking process at grain boundaries under high temperature and low stress [9]. A number of Continuum Damage Mechanics (CDM) based constitutive models have been developed for creep damage prediction [10]. An early and often cited attempt came from Kachanov [7] and Rabotnov [8] in the form of coupled creep strain rate and damage evolution equations for isotropic materials as follows

$$\dot{\epsilon}_{cr} = \frac{d\epsilon_{cr}}{dt} = A\left(\frac{\overline{\sigma}}{1-\omega}\right)^n \tag{1}$$

$$\dot{\omega} = \frac{d\omega}{dt} = \frac{M\overline{\sigma}^{\chi}}{(1-\omega)^{\phi}} \tag{2}$$

where A and n are secondary creep constants, M, χ, and ϕ are tertiary creep damage constants, and $\dot{\epsilon}_{cr}$ and $\overline{\sigma}$ are the equivalent creep strain and von Mises stress respectively [11–13]. This isotropic formulation regards damage as a scalar state variable, ω that accounts for all microstructural degradations exhibited in a material. Numerous specialized variations on this isotropic constitutive model can be found throughout literature [14–20]. The constitutive model has also been generalized for multiaxial states of stress in isotropic materials using elastic compliance tensors and the stress deviator [21].

Literature shows that under creep conditions, intergranular damage must be represented by multiple principal damage variables. Damage can induce an anisotropic creep response [5,22]. Literature shows that damage anisotropy can be indentified in two material classes, aluminum-like and copper-like [9,23]. For aluminum-like materials, damage is typically distributed isotropically. Aluminum-like materials with a simple stress state (i.e., uniaxial creep tests) can be modeled with isotropic creep damage models [22–25]. For copper-like materials, damage develops mainly on the plane perpendicular to the first principal stress. Copper-like materials and components undergoing a complex state of stress exhibit damage induced anisotropic creep response which must be accounted for with more robust modeling techniques [26]. Models have been developed that can account for both aluminum-like and copper-like materials, and the range of intermediate behaviors between them [5,27]. These isotropic constitutive models are unable to model anisotropic microstructure materials.

A number of researchers have developed constitutive models for transversely-isotropic materials; however, most formulations do not accurately model intermediate material orientations (when the longitudinal grains are not parallel or perpendicular to the load direction) [3,28,29]. For anisotropic microstructure materials, the effect of material orientation must be included in the both the creep strain rate and damage evolution equations [30].

3. Constitutive model

In order to produce an accurate multiaxial representation of the creep deformation of a transversely-isotropic material it is first necessary to accurately model the uniaxial x_1-x_2 plane of symmetry and x_3 normal represented by T and L specimen, respectively. Isotropic creep damage models are commonly implemented for simple cases involving uniaxially loaded isotropic materials. First the framework for a secondary creep model for transversely-isotropic materials based on Norton's power law is outlined. Next, an anisotropic tertiary creep damage constitutive model for transversely-isotropic materials based on Kachanov-Rabotnov is given.

3.1. Secondary creep constitutive model

For anisotropic creeping materials, if creep strain rate tensor$\dot{\epsilon}_{ij}$, and Cauchy stress tensor, σ_{ij}, follow Norton's power law, the creep behavior can be expressed as follows

$$\dot{\epsilon}_{ij} = A_{ij}\sigma_{ij}^n \tag{3}$$

where it is assumed that the creep exponent, n, has the same value in all anisotropic principal directions [31]. Using the appropriate equivalent stress function this can be reduced to an equivalent strain function similar to Eq. (1).

$$\dot{\epsilon}_{cr} = A_{aniso}\left[\tilde{q}(\sigma_{ij})\right]^n \tag{4}$$

where A_{aniso} is an equivalent creep coefficient of the anisotropic material and $\tilde{q}(\sigma_{ij})$ is equivalent deviatoric stress function. For isotropic creeping materials, A_{aniso} and $\tilde{q}(\sigma_{ij})$ become the isotropic creep coefficient under multiaxial stress state, A, and the equivalent stress, σ_e. In this paper the discussion is confined to a case where the Cartesian coordinate axes coincide with the symmetry axes of creep orthotropy. For a material of creep orthotropy, the equivalent deviatoric stress function $\tilde{q}(\sigma_{ij})$ can be defined as Eq. (5) in analogy to Hill's yield function [32].

$$\tilde{q}(\sigma_{ij}) = \Big[F(\sigma_{22}-\sigma_{33})^2+G(\sigma_{33}-\sigma_{11})^2+H(\sigma_{11}-\sigma_{22})^2+2L\sigma_{23}^2 +2M\sigma_{13}^2+2N\sigma_{12}^2\Big]^{1/2} \tag{5}$$

where, F, G, H, L, M, and N are unitless material constants which describe the current state of creep orthotropy, and σ_{11}, σ_{22}, etc. are stress components. During creep deformation of a material, the state of anisotropy changes; however, it will be assumed that the change in anisotropy is negligible compared with the initial state of anisotropy, as assumed in the anisotropic plastic theory [32]. Namely, the state of anisotropy remains constant. Hence, by substituting Eq. (5) into Eq. (4) and solving for the cases of principal axes of orthotropy ($i = j = 1, 2,$ and 3) and shear directions ($i = 2$ and $j = 3$, $i = 3$ and $j = 1$, and $i = 1$ and $j = 2$), expressions for the creep orthotropy parameters can be obtained as shown in Eq. (6) in terms of creep coefficient ratios. In deriving Eq. (6), the equivalent deviatoric stress function in shear directions of $\tau_e = \sigma_e/\sqrt{3}$ was applied as in the isotropic case.

$$\begin{aligned}
F &= \frac{1}{2}\left\{\left(\frac{A_{22}}{A_{aniso}}\right)^{2/n}+\left(\frac{A_{33}}{A_{aniso}}\right)^{2/n}-\left(\frac{A_{11}}{A_{aniso}}\right)^{2/n}\right\},\\
G &= \frac{1}{2}\left\{\left(\frac{A_{33}}{A_{aniso}}\right)^{2/n}+\left(\frac{A_{11}}{A_{aniso}}\right)^{2/n}-\left(\frac{A_{22}}{A_{aniso}}\right)^{2/n}\right\},\\
H &= \frac{1}{2}\left\{\left(\frac{A_{11}}{A_{aniso}}\right)^{2/n}+\left(\frac{A_{22}}{A_{aniso}}\right)^{2/n}-\left(\frac{A_{33}}{A_{aniso}}\right)^{2/n}\right\},\\
L &= \frac{3}{2}\left(\frac{A_{23}}{A_{aniso}}\right)^{2/n}, M = \frac{3}{2}\left(\frac{A_{13}}{A_{aniso}}\right)^{2/n}, \text{and} N = \frac{3}{2}\left(\frac{A_{12}}{A_{aniso}}\right)^{2/n}.
\end{aligned} \tag{6}$$

Assume that the orthotropic creeping material has identical properties in both transverse directions. This transversely-isotropic material has three independent stress components. In order to derive these stress components mathematically, a plane consisting of axis 1 and 2 which is perpendicular to axis 3 is defined as an isotropic plane. Derivation of theses stress components is similar to that of the orthotropic plastic material [32]. The following relationship among the parameters can be obtained,