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Mechanical and thermal stresses in a functionally graded rotating disk with variable thickness due to radially symmetry loads

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ABSTRACT

Rotating disks have many applications in the aerospace industry such as gas turbines and gears. These disks normally work under thermo mechanical loads. Minimizing the weight of such components can help reduce the overall payload in aerospace industry. For this purpose, a rotating functionally graded (FG) disk with variable thickness under a steady temperature field is considered in this paper. Thermo elastic solutions and the weight of the disk are related to the material grading index and the geometry of the disk. It is found that a disk with parabolic or hyperbolic convergent thickness profile has smaller stresses and displacements compared to a uniform thickness disk. Maximum radial stress due to centrifugal load in the solid disk with parabolic thickness profile may not be at the center unlike uniform thickness disk. Functionally graded disk with variable thickness has smaller stresses due to thermal load compared to those with uniform thickness. It is seen that for a given value of grading index, the FG disk having concave thickness profile is the lightest in weight whereas the FG disk with uniform thickness profile is the heaviest. Also for any given thickness profile, the weight of the FG disk lies in between the weights of the all-metal and the all-ceramic disks.

1. Introduction

Functionally graded materials (FGMs) are those in which the volume fraction of the two or more materials is varied continuously as a function of position along certain dimension(s) of the structure [1,2]. These materials which are mainly constructed to operate in high temperature environments find their application in aerospace, turbine rotors, flywheels and gears just to mention a few. As the use of FGMs increases, new methodologies need to be developed to characterize, analyze and design structural components made of these materials.

FGMs are usually made of a mixture of ceramic and metals. The ceramic constituent of the material provides the high temperature resistance due to its low thermal conductivity. The ductile metal constituent, on the other hand, prevents fracture caused by stress due to high temperature gradient in a very short period of time [3].

The rotating disks subjected to mechanical and thermal loads have been studied in both linear and nonlinear forms. In linear analysis, researchers mainly used infinitesimal elasticity theory (see e.g. [4]) for the study of isotropic or anisotropic disks of uniform thickness profiles. In nonlinear case, the scientific literature mainly focused on three aspects namely nonlinear geometry, material and analysis.

Although many earlier studies on rotating disks (see e.g. [5] and the references there in) considered disks with uniform thickness, later several authors (see e.g. [6–9]) considered the nonlinear geometry of the rotating disks emphasizing the importance of the variable thickness. Recent studies [10,11] indicated that stresses in rotating disks (annular or solid) with variable thickness were much lower than those in a uniform thickness disk at the same angular velocity. Unlike these studies where disks were subjected to mechanical loads only, many studies can be seen in the literature (see e.g. [12–16]) with disks subjected to thermal load only. Literature also contains studies on FGM disks subjected to thermo mechanical loading.

Ruhi et al. [17] presented a semi-analytical solution for thick-walled finitely-long cylinders made of FGMs under thermo mechanical load. Fukui and Yamanaka [18] studied the effects of the gradation of components on the strength and deformation of thick-walled FG tubes under mechanical load such as internal pressure

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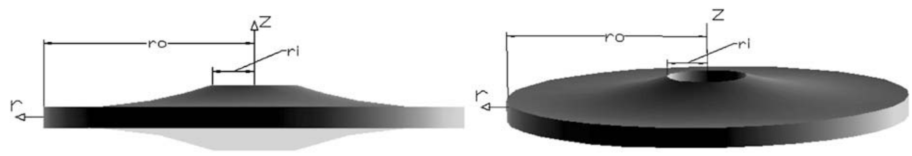

Fig. 1. Configuration of a thin disk with variable thickness.

with plane strain conditions. Fukui et al. [19] extended their previous work by considering a thick-walled FG tube under uniform thermal loading. They investigated the effect of graded components on residual stresses.

In recent years, application of non-homogeneous material such as FGMs in rotating disks increased and many studies focused on FG rotating disks of uniform thickness with thermo mechanical loading. Durodola and Attia [20,21] provided a finite element analysis for FG rotating disks using commercial software. Considering disks of non-homogeneous orthotropic materials such as those obtained through non-uniform reinforcement of metal matrix by long fibers, they considered three types of gradation distribution of the Young's modulus E in the hoop direction relative to matrix material modulus. Kordkheili and Naghdabadi [22] presented a semi-analytical thermo elastic solution for hollow and solid rotating axisymmetric disks made of functionally graded materials under plane stress condition. They compared their results with Durodola and Attia [20,21] under the centrifugal loading.

Jahed and Sherkatti [23] applied the variable material properties (VMP) method to obtain stresses in an inhomogeneous rotating disk with variable thickness under steady temperature field assuming the material properties as field variables. The spatial distributions of field variables were found in an iterative manner. Jahed and Shirazi [24] used VMP method to evaluate the temperature in a rotating disk during heating and cooling. Farshi et al. [25] used VMP method to obtain an optimum profile of an inhomogeneous non-uniform rotating disk. Jahed et al. [26] analyzed an inhomogeneous disk model to achieve minimum weight of the disk with variable thickness. Using the VMP method, stresses were obtained in the rotating disk under a steady temperature field.

In the present paper, a thin FG disk of variable thickness (Fig. 1) subjected to centrifugal body and thermal loading is considered. Plane stress condition and the symmetry with respect to the axis and the mid-plane are assumed. The main aim of this investigation is to enhance the understanding of the elastic behavior of hollow and solid FG disks subjected to thermo mechanical loading and relate this behavior to some basic factors such as material property gradation and the geometry of the disks under appropriate (free–free or fixed–free type) boundary conditions. This study also investigates the question how do grading and geometry of the disks relate to their weight and hence to their optimum design. Based on the form of the power-law distribution for the mechanical properties of the constituent components and the thickness profile function, semi-analytical method is employed in this paper to obtain the thermo elastic solutions for the non-dimensional temperature distribution and the displacement field in the disks.

In semi-analytical method, the radial domain of the disk is divided into some virtual sub-domains where, in each sub-domain, the mechanical property is assumed to be constant. This assumption yields the governing equilibrium equations in each sub-domain as ordinary differential equations with constant coefficients whose general solution can be written involving certain unknowns. These unknowns can be determined as solution of system of linear algebraic equations obtained by imposing the continuity conditions at the interface of the adjacent sub-domains together with global conditions. Increasing the number of sub-domains (divisions) in the radial direction increases the accuracy in the solution.

2. Gradation relation

In this study, the property variation P of the material in the FG disk along the radial direction is assumed of the following form [17,22,27]:

$$P(r) = (P_o - P_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n + P_i; \quad r_i < r < r_o \tag{1a}$$

Here P_o and P_i are the corresponding properties of the outer and inner faces of the disk; r_o and r_i are the outer and inner radii of the disk, respectively; $n \geq 0$ is the volume fraction exponent (also called grading index in this paper). The power-law for Eq. (1a) is widely accepted and reflects a simple rule of mixtures in terms of the volume fraction of the materials. This study assumes Poisson's ratio ν to be constant and the elastic modulus E, thermal conductivity K, thermal coefficient of expansion α and density ρ to be varied according to the gradation relation of Eq. (1a), for example, the assumed form for the modulus of elasticity E is:

$$E(r) = (E_o - E_i)\left(\frac{r - r_i}{r_o - r_i}\right)^n + E_i; \quad r_i < r < r_o \tag{1b}$$

It may be noted that the same grading index n has been used in this study while defining the material properties for the modulus of elasticity, thermal conductivity, thermal coefficient and the density.

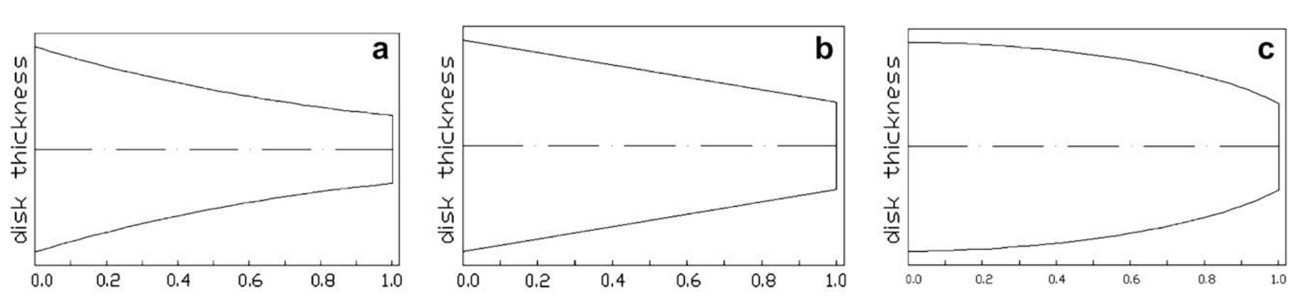

Fig. 2. Thickness profiles of FG disk (a) concave (b) linear and (c) convex.