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The dynamics of a caterpillar drive☆

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ABSTRACT

The steady-state vertical oscillations of an infinite horizontal periodic chain of identical uniform solid rods joined by hinges and supported by a uniform viscoelastic base are studied. The oscillations are excited by a vertical complex harmonic force that moves along the chain at a constant speed. They are assumed to be steady oscillations in the following sense: a shift along the chain by a distance equal to the length of a rod causes the complex vertical deflection of the chain to be delayed for a time, during which the complex harmonic force is displaced by this distance. The problem is solved using a Fourier transform under the assumption that each point of the chain is in a quiescent state long before the approach of the harmonic force, and returns to the same state under the action of the viscous forces of the base after the harmonic force has withdrawn to infinity.

The motion of heavy caterpillar vehicles gives rise to vibrations of the ground, generating surface waves that propagate over a long distance, causing damage to buildings. If the passage of these vehicles is prohibited, seismographs are set up at the place of prohibition that record any infringement of the ban. As a result, the waves generated by caterpillar vehicles have been extensively studied in recent years.[1–5] A simplified model of a drive was used but the dynamic properties of a caterpillar track that has a periodic structure were not taken into consideration.

The majority of vehicles with caterpillar tracks have two motors (Fig. 1) with independent drive mechanisms.[4] Each of them is driven through a toothed wheel at the rear and, in the case of curved motion, they have a different speed. The weight of the vehicle is transferred onto each caterpillar drive through a number of wheels located close to one another. The ground vibrations caused by the motion of the vehicle are therefore quite complex.

1. A dynamic model of a caterpillar track. Natural oscillations

We shall consider the dynamic phenomena associated with the periodic structure of a caterpillar track. The deformation of the ground is considerably greater than the deformation of a link of the caterpillar track, since the distance between neighbouring hinges of the track is small. A link of a caterpillar track can therefore be treated as a rigid body. With the aim of simplifying the study, we shall neglect the interaction of the wheels through the caterpillar track as well as the interaction of the two caterpillar tracks through the ground.

In the case of the motion of a vertical harmonic force along a uniform or periodic structure, supported on the ground, with a velocity close to the velocity of elastic surface waves in the ground, loss of stability of the motion is accompanied by an exponential increase in the oscillations.[6,7] The dynamic phenomena studied earlier are observed at a velocity of the motion of the tracked vehicle that is considerably less than the propagation velocity of surface waves in soft ground. The ground can therefore be represented in a simplified way as a uniform viscoelastic foundation. This approach has been used in the analysis of the dynamics of a railway track.[8,9]

Note that, in some studies, pontoon crossings are treated as a chain of rods joined by hinges and supported on a uniform elastic foundation,[10] which simulates the hydrostatic pressure on pontoons. The oscillations of a periodic chain of rigid bodies under the action of a fixed harmonic force have been studies.[11]

We will represent the caterpillar track in the form of an infinite horizontal chain of identical uniform solid bars of length l joined by hinges. The linear density of the chain ρ was calculated using the mass of the rods, the hinges and the ground, that vibrates together with the caterpillar track. The chain is supported by a uniform viscoelastic inertialess foundation, with which the chain is in continuous contact,

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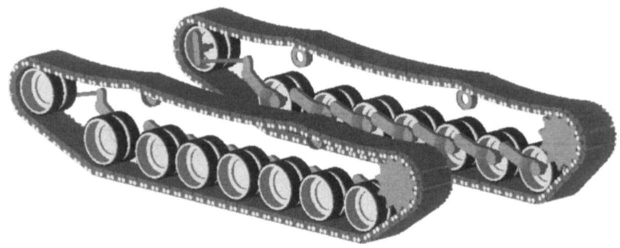

Fig. 1.

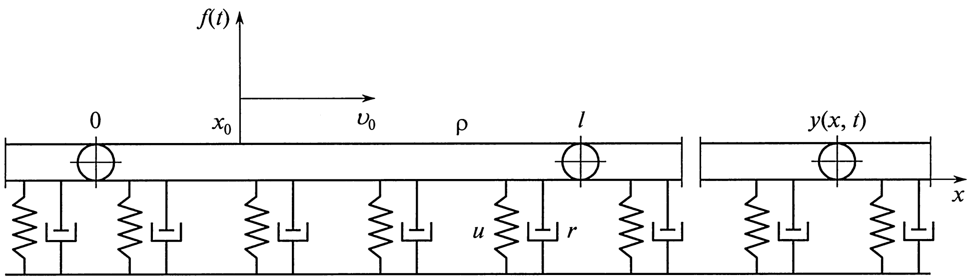

Fig. 2.

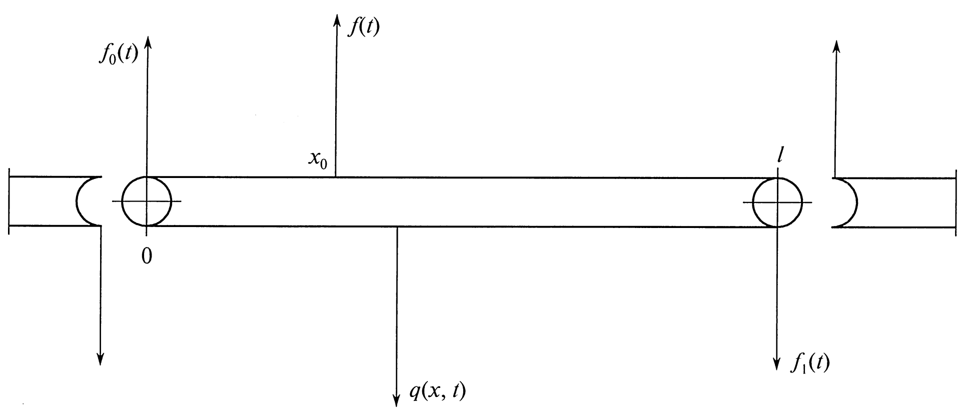

Fig. 3.

and u and r are the elastic stiffness and viscosity of the foundation (Fig. 2). An idle caterpillar track chain in the quiescent state gives rise to a uniform deformation of the foundation.

We shall measure the vertical deviations of the chain $y(x,t)$ from its static equilibrium state. Suppose x is the longitudinal coordinate and the value $x=0$ corresponds to one of the hinges. The deviation of a link of the chain is completely determined by the deviations of the hinges at the ends of this link. The deviation of the link in Fig. 2 has the form

$$y(x,t) = y(0,t)\frac{l-x}{l} + y(l,t)\frac{x}{l} \tag{1.1}$$

The distributed load

$$q(x,t) = \delta y(x,t), \quad \delta = \rho\frac{\partial^2}{\partial t^2} + r\frac{\partial}{\partial t} + u \tag{1.2}$$

that includes the force of inertia of the chain and the reaction of the foundation, is applied to the chain and its positive direction is shown in Fig. 3. If there are no external forces, the forces of the interaction between neighbouring links of the chain through the hinges are equal to zero and the natural oscillations of the chain satisfy the linear homogeneous equation $q(x,t)=0$, which has a particular solution

$$y(x,t) = a\exp(-\mu t)\sin\left(\sqrt{\omega_e^2-\mu^2}t\right), \quad \omega_e = \sqrt{\frac{u}{\rho}}, \quad \mu = \frac{r}{2\rho} \tag{1.3}$$

The viscosity of the foundation r is usually small and satisfies the condition

$$r < 2\sqrt{\rho u} \tag{1.4}$$

The treatment will henceforth be limited to the case when condition (1.4) is satisfied. In this case, the solution (1.3) is real and describes the decaying oscillations of the chain as a rigid body. The vertical deviations of all points of the chain are identical.