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The spatial contact problem for an elastic layer and wavy punch when there is friction and wear☆

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ABSTRACT

The spatial (three-dimensional) problem of the wear of a wavy punch sliding over an elastic layer bonded to a rigid base, assuming there is complete contact between the punch and the layer, is considered. It is assumed that there is Coulomb friction and wear of the punch. An analytical expression for the contact pressure is constructed using the general Papkovich–Neuber solution, the harmonic functions in which are represented in the form of double Fourier integrals, after which the problem reduces to a linear system of differential equations. It is established that the harmonics constituting the shape of the punch and the contact pressure are shifted with respect to one another in time along the sliding line of the punch. The velocity of this shift depends on the longitudinal and transverse frequencies of the harmonic, that is, dispersion of the waves is observed.

The solution of the periodic contact problem in the theory of elasticity was given for the first time for a system of punches with plane bases in contact with an elastic half plane when there was no friction.[1] Later, the case of the periodic contact of smooth punches was considered,[2,3] and sliding friction in the contact was taken into account[4] as well as the possibility of adhesion and wear of the contacting surfaces.[5] A fairly full description of the different formulations of periodic contact problems in the theory of elasticity and methods for solving them is available.[6]

Contact problems for wavy bodies can be subdivided into two groups, that is, with complete and partial contact of the bodies. Homogeneous boundary conditions, that enable exact solutions of the corresponding contact problems, including spatial problems,[7,8] to be obtained, correspond to complete contact. Problems with partial contact belong to a more complex class of mixed boundary value problems. Their analytical solution is only possible when certain simplifying assumptions are made such as, for example, in the case of weak loading.[9]

The spatial problem of the complete contact of a wavy punch with an elastic layer is considered below. Its formulation, unlike the spatial contact problems mentioned above, assumes that there is friction and wear of the punch. An exact solution of this problem is constructed.

1. Statement of the problem and basic equations

Consider the sliding coupling of an elastic layer of thickness h, bonded to an absolutely rigid plane base, and an absolutely rigid body (the punch) in contact with the layer over the whole of its surface (Fig. 1). The elastic properties of the layer are specified by the shear modulus G and Poisson's ratio ν. We introduce the system of coordinates $Oxyz$, in which the Oxy plane coincides with the boundary of the base and the z axis passes through a certain fixed point S of the punch.

Suppose the base together with the layer moves with a velocity V in a direction opposite to the x axis while the system of coordinates $Oxyz$ remains in place and the punch can only be displaced along the z axis (Fig. 1). As a result of the punch sliding over the layer, friction

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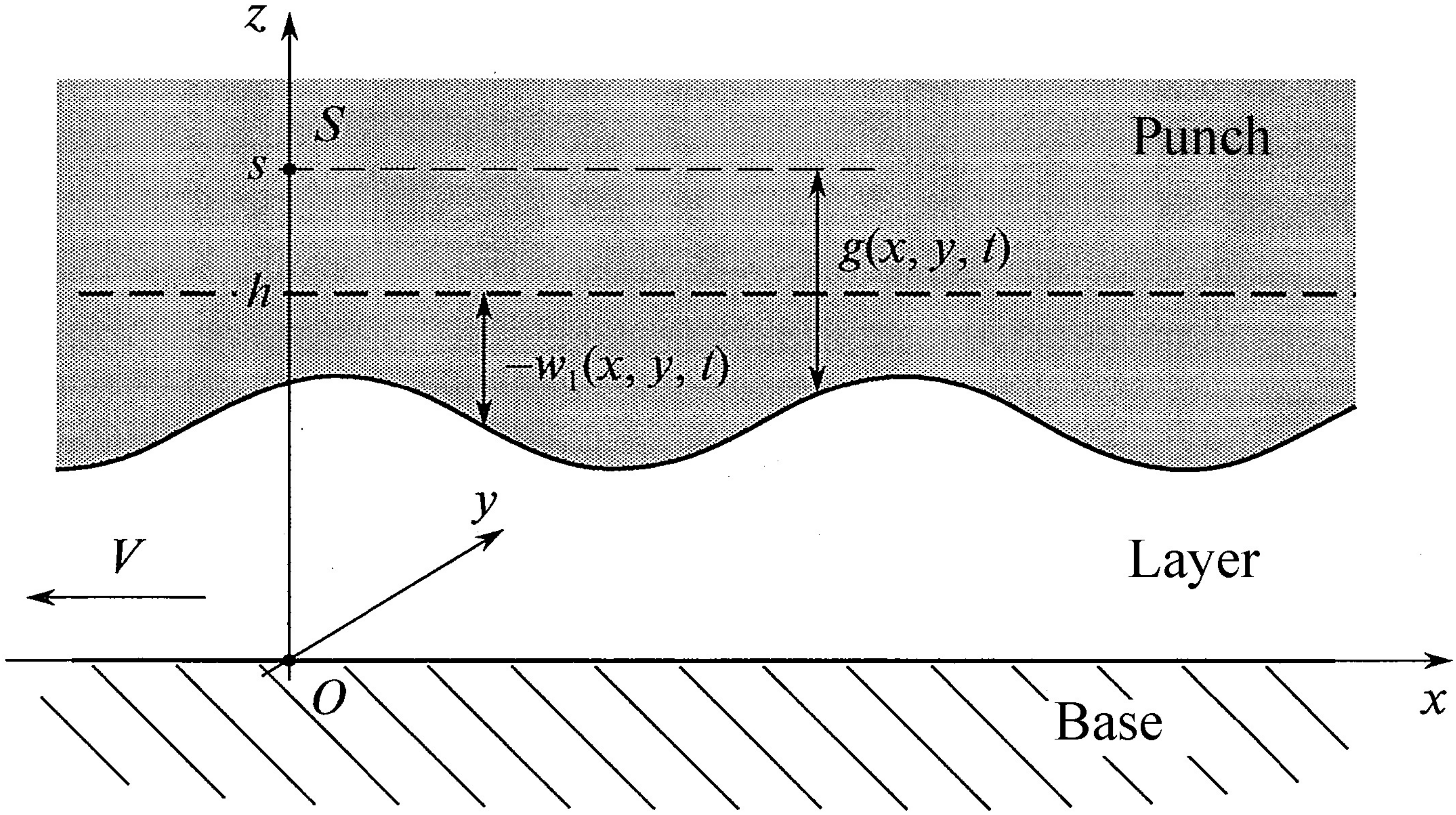

Fig. 1.

occurs and there is wear of the punch. We will assume that the boundary shearing stresses τ generated by the friction are related to the contact pressure p by Coulomb's law[10]

$$\tau = \mu p + \tau_0; \quad \tau = \tau_{zx}\big|_{z=h}, \quad p = -\sigma_z\big|_{z=h} \tag{1.1}$$

where μ is the coefficient of sliding friction and τ_0 is the adhesion component of the friction. The generally accepted notation for the components of a stress-strain state[11] will be used henceforth.

The local wear of the punch W is determined by the corresponding contact pressure according to the elementary wear law[10]

$$\partial W(x, y, t)/\partial t = k_w p(x, y, t) \tag{1.2}$$

where k_w is a wear-resistance parameter that, like the coefficient of friction μ, can depend on the sliding velocity V. It is henceforth assumed that $x, y, \in(-\infty, \infty)$ and the instant $t = 0$ is taken as the start of the wear.

The condition for complete contact of the punch with the layer (over the whole of its surface) implies that the contact pressure is positive:

$$p(x, y, t) > 0 \tag{1.3}$$

Inequality (1.3) must be checked for the contact pressure distributions obtained.

The z coordinate of the point S is denoted by s. We shall use the distance $g(x, y, t)$ between the contact surface of the punch and the plane $z = s(t)$ to describe the shape of the punch, which changes as it wears (Fig. 1). The shape of the punch is assumed to be mildly sloping (henceforth corresponding partial derivatives are denoted by the subscripts x and y on a function symbol)

$$|g_x(x, y, t)| \ll 1, \quad |g_y(x, y, t)| \ll 1 \tag{1.4}$$

This, in particular, enables the wear W to be identified with the displacement of the contact surface of the punch along the z axis as a result of its wear.

Regarding the shape $g(x, y, t)$ of the punch, we will assume that it is a set of N harmonics (waves) of different frequencies (henceforth summation is carried out everywhere from $n = 1$ to $n = N$)

$$g(x, y, t) = \bar{g}(t) + U(x, y, t) \tag{1.5}$$

$$U(x, y, t) = \sum A_n(t)\sin(\omega_{xn}x + \rho_{xn}(t))\sin(\omega_{yn}y + \rho_{yn}(t))$$
$$\equiv \sum(a_n(t)c_{xn}c_{yn} + b_n(t)s_{xn}c_{yn} + c_n(t)c_{xn}s_{yn} + d_n(t)s_{xn}s_{yn}) \tag{1.6}$$