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Prediction of stiffness degradation in hygrothermal aged $[\theta_m/90_n]_S$ composite laminates with transverse cracking

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ABSTRACT

The stiffness reduction of symmetric hygrothermal aged angle-ply laminates containing a cracked mid-layer is predicted by using a modified shear-lag model. The material properties of the composite are affected by the variation of temperature and moisture, and are based on a micro-mechanical model of laminates. This hygrothermal effect is taken into account to assess the changes in the longitudinal modulus due to transverse cracking. The obtained results represent well the dependence of the degradation of elastic properties on the cracks density and hygrothermal conditions.

1. Introduction

Matrix cracking in laminated composites has been extensively investigated in the literature both experimentally and theoretically. Stress analysis methods based on shear-lag arguments (Parvizi and Bailey, 1978; Highsmits and Reifsnider, 1982; Smith and Wood, 1990; Laws and Dvorak, 1988; Lim and Hong, 1989a,b; Lee and Daniel, 1990; Berthelot, 1997) and variational principles (Hashin, 1985; Varna and Berglund, 1991; Nairn, 1989) were proposed and are now available, by incorporating the homogenization procedure in order to derive the expressions for the stiffness reduction. Recently, Tounsi et al. (Tounsi and Amara, 2005; Tounsi et al., 2005; Amara et al., 2005) have studied the stiffness reduction in hygrothermal aged cross-ply laminates using a modified shear-lag models by introducing the stress perturbation function. It is well known that during the operational life, the variation of temperature and moisture reduces the elastic moduli and degrades the strength of the laminated material (Shen et al., 1981; Adams and Miller, 1977; Bowles and Tompkins, 1989; Shen, 2001; Megueni et al., in press; Sereir et al., 2005, 2006).

In this paper, a complete parabolic shear-lag model (Berthelot, 1997) is used with some modifications to predict the effect of transverse cracks on the stiffness degradation of hygrothermal aged angle-ply composite laminates. General expression for longitudinal modulus reduction versus transverse crack density is obtained by introducing the stress perturbation function (Tounsi and Amara, 2005; Amara et al., 2005; Joffe and Varna, 1999). Good agreement is obtained by comparing prediction with experimental results. Then, the hygrothermal effect on the material properties of the laminate is taken into account to evaluate the stiffness loss in angle-ply

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laminates containing transverse cracks. The obtained results illustrate well the dependence of the degradation of elastic properties on the cracks density, hygrothermal conditions and the fibre orientation of the outer layers.

2. Theoretical analysis

It is well known in many studies (Shen et al., 1981; Adams and Miller, 1977; Bowles and Tompkins, 1989; Shen, 2001; Megueni et al., in press; Sereir et al., 2005, 2006) that the material properties are function of temperature and moisture. In terms of a micro-mechanical model of laminate, the material properties may be written as (Tsai and Hahn, 1980):

$$E_L = V_f E_f + V_m E_m \quad (1)$$

$$\frac{1}{E_T} = \frac{V_f}{E_f} + \frac{V_m}{E_m} - V_f V_m \frac{\nu_f^2 (E_m/E_f) + \nu_m^2 (E_f/E_m) - 2\nu_f \nu_m}{V_f E_f + V_m E_m} \quad (2)$$

$$\frac{1}{G_{LT}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \quad (3)$$

$$\nu_{LT} = V_f \nu_f + V_m \nu_m \quad (4)$$

In the above equations, V_f and V_m are the fibre and matrix volume fractions and are related by

$$V_f + V_m = 1 \quad (5)$$

E_f, G_f and ν_f are the Young's modulus, shear modulus and Poisson's ratio, respectively, on the fibre, and E_m, G_m and ν_m are the corresponding properties for the matrix.

It is assumed that E_m is a function of temperature and moisture, as is shown in Section 3.2, then E_L, E_T and G_{LT} are also functions of temperature and moisture.

2.1. *Stiffness reduction model in the angle-ply laminates*

Transverse matrix cracking is a common damage mode in angle-ply laminates under uniaxial tension. The matrix cracks develop in the fibre direction and extend across the 90°-ply width from the free edges.

Consider the idealised angle-ply laminates shown in Fig. 1. When such laminate is loaded in uniaxial tension the first damage which occurs is transverse cracking in the middle layer. The spacing between cracks is assumed to be equidistant, which means that laminate contains a periodical array of

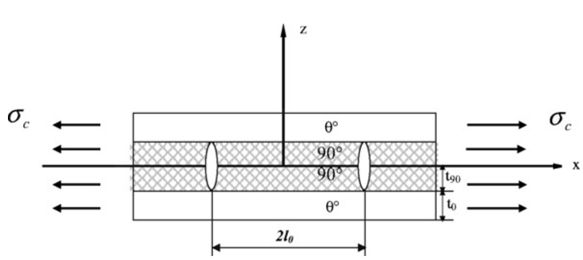

Fig. 1 – Transverse cracked angle-ply laminate and geometric model.

cracks in 90° layer. The geometry of the repeatable unit used for modelling is shown in Fig. 1. Dimensionless coordinates can be introduced:

$$\bar{z} = \frac{z}{t_{90}}; \quad \bar{l}_0 = \frac{l_0}{t_{90}}; \quad \alpha = \frac{t_\theta}{t_{90}}; \quad \bar{x} = \frac{x}{t_{90}}; \quad h = t_\theta + t_{90} \quad (6)$$

Loading is applied only in x-direction and the far field applied stress is defined by $\sigma_c = (1/2h)N_x$, where N_x is applied load.

The following analysis will be performed assuming generalized plane strain condition:

$$\varepsilon_y^\theta = \varepsilon_y^{90} = \varepsilon_y = \text{const} \quad (7)$$

The symbol (bar) over stress and strain components denotes volume average. They are calculated using the following expressions:

(a) in the θ° layer:

$$\bar{f}^\theta = \frac{1}{2l_0}\frac{1}{t_\theta}\int_{-l_0}^{+l_0}\int_{t_{90}}^{h} f^\theta \, dx\, dz = \frac{1}{2\bar{l}_0}\frac{1}{\alpha}\int_{-\bar{l}_0}^{+\bar{l}_0}\int_{1}^{\bar{h}} f^\theta(\bar{x},\bar{z})\, d\bar{x}\, d\bar{z} \quad (8)$$

(b) in 90° layer:

$$\bar{f}^{90} = \frac{1}{2l_0}\frac{1}{t_{90}}\int_{-l_0}^{+l_0}\int_{0}^{t_{90}} f^{90} \, dx\, dz = \frac{1}{2\bar{l}_0}\int_{-\bar{l}_0}^{+\bar{l}_0}\int_{0}^{1} f^{90}(\bar{x},\bar{z})\, d\bar{x}\, d\bar{z} \quad (9)$$

By using the strains in the θ° layer (which is not damaged and, hence, strains are equal to laminate strains, $\varepsilon_x = \bar{\varepsilon}_x^\theta$, etc.) and assuming that the residual stresses are zero, the Young's modulus of the laminate with cracks may be defined from following expression:

$$E_x = \frac{\sigma_c}{\bar{\varepsilon}_x^\theta} \quad (10)$$

Note that the initial laminate modulus measured at the same load is $E_{x0} = \sigma_c/\varepsilon_{x0}$ and, hence

$$\frac{E_x}{E_{x0}} = \frac{\varepsilon_{x0}}{\bar{\varepsilon}_x^\theta} \quad (11)$$

2.2. *Averaged stress–strain relationships for laminates with cracks*

Constitutive equations that give the relationship between strain and stresses are:

(a) in the 90° layer:

$$\begin{Bmatrix} \varepsilon_x^{90} \\ \varepsilon_y^{90} \\ \varepsilon_z^{90} \end{Bmatrix} = \begin{bmatrix} S_{22} & S_{12} & S_{23} \\ S_{12} & S_{11} & S_{12} \\ S_{23} & S_{12} & S_{22} \end{bmatrix} \begin{Bmatrix} \sigma_x^{90} \\ \sigma_y^{90} \\ \sigma_z^{90} \end{Bmatrix} \quad (12)$$