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On The Semi-Analytical Solution of Integro-Partial Differential Equations

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Abstract

The breakage and aggregation processes in batch systems had attained highly interest in applied mathematics and engineering fields. In this work, we developed analytical solutions of the particle breakage and aggregation using the population balance equation (PBE) in batch flow systems. To allow explicit solutions, we approximated the particle breakage and aggregation mechanisms by assuming functional forms for breakage and aggregation kernels. In this framework, the Adomian decomposition method (ADM), variational iteration method (VIM) and homotopy perturbation method (HPM) were used to solve the population balance model. These semi-analytical methods overcome the crucial difficulties of numerical discretization and stability that often characterize previous solutions of the PBEs. The obtained results in all cases showed that the predicted particle size distributions converge exactly in a continuous form to that of the analytical solutions using the three methods.

Keywords: Population balance model; Adomian decomposition method; Variational iteration method; Homotopy perturbation method

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1. Introduction

Nonlinear integro-partial differential equations are encountered in various fields in applied mathematics[1,2] and engineering[3]. For example, population balance equation (PBE) is used to describe various kinds of phenomena such as crystallization[4,5], polymerization[6-8], aerosol[9], granulation[10–13], and biological [15] .
The continuous distribution of particles for PBE in batch system is usually described by a number density function $n(v,t)$, which represents the number of particles within a differential volume size range per unit volume of latex. The rate of change of the particle number density is described by a nonlinear integro-partial differential population balance equation for one-dimensional[16-18]

$$\frac{\partial n(v,t)}{\partial t} = \int_v^{\infty} \beta(v/v')\Gamma(v')n(v',t)dv' + \frac{1}{2}\int_0^{v} \omega(v-v',v')n(v-v',t)n(v',t)dv' - \Gamma(v)n(v,t) - \int_0^{\infty} \omega(v,v')n(v,t)n(v',t)dv'. \tag{1}$$

with the initial condition $n_0(0,v) = n_0(v) \geq 0.$

Here it is assumed that at any instant the number of particles $n(v,t)$ depends only on time and one internal coordinate, which is the particle volume v .
where: $\Gamma(v)$ and $\omega(v,v')$ are the breakup and aggregation frequencies, respectively, and $\beta(v/v')dv$ is the number of daughter particles having a volume in the range from v to $v+dv$ formed upon breakup of the particle of volume v' . The first two terms on the right hand side represent particle formation due to breakup and aggregation succeeded by two terms which represent particle loss due to breakup and aggregation.
Numerical solution of the above PBE (1) is difficult due to the integral and the non-linear behavior of the source term. There are only few numerical techniques available in the literature to compute the complete property distribution. Some numerical techniques can be found in the following literature [19-25], these series of papers on the available numerical methods were discussed up to the mid-eighties to find efficient and stable numerical methods for solving the population balance equation, such as the fixed- and moving pivot methods, Dual Quadrature Method of Generalized Moments (DuQMoGeM), and Cumulative Quadrature Method of Moments(CQMOM).
In recent years, several methods have drawn particular attention, such as the Adomian decomposition method[26], the variational iteration method [27], and the homotopy perturbation method[28]. The main advantage of the techniques are the most transparent methods of solution of (PBEs) and can be coupled with an optimization algorithm to solve inverse problems of the estimation of properties of building materials because they provide immediate and visible symbolic terms of both analytical and numerical solutions to linear as well as nonlinear integro-partial differential equations without linearization or discretization.
We propose in this work these three semi analytical methods as a novel converging sequence of continuous approximations to the particle number density function with new breakage and aggregation kernels, which a solution to the PBE.

Nomenclature

$n(v,t)\,dv$	Number of particles in size range
$n_m(v,t)$	Solution components
v, v'	Particle volumes