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Peristaltic flow of a Sisko fluid over a convectively heated surface with viscous dissipation

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ABSTRACT

We have analyzed the peristaltic flow of the mathematical model for a non-Newtonian Sisko fluid in the presence of a magnetic field and the heat transfer problem with the effects of both variable thermal conductivity and viscous dissipation. The governing equations of a non-Newtonian fluid along with heat and nanoparticles are modeled and simplified by the assumption of a low Reynolds number and a long wavelength. The velocity equation is solved by use of the homotopy perturbation technique, while the exact solutions are computed for temperature and concentration equations. The solutions depend on the Brinkman number (B_κ) and magnetohydrodynamics (M). The expressions obtained for the velocity, temperature, and concentration profiles are plotted, and the impact of various physical parameters is investigated for different peristaltic waves. We found that temperature, concentration, and pressure gradient are increasing functions of the Sisko parameter b, Brinkman number (B_κ), and magnetohydrodynamics (M), respectively.

1. Introduction

The process of symmetrical contraction and expansion of progressive waves on the walls of a channel that mixes and transports fluid in a channel is termed *peristalsis*. In several procedures in physiology and engineering, peristaltic flow concerns are broadly encountered in a channel or tube. The applications of peristalsis cover swallowing food through the esophagus, transportation of urine from the kidney to the bladder, propulsion of chyme in the gastrointestinal tract, ovum movement in the female fallopian tube, vasomotion of narrow blood vessels, movement of spermatozoa in the human reproductive tract, and movement of water from the ground to the top branches of trees [1].

Peristaltic flows have many biological and industrial applications, such as the blood pump in the heart, lung machines, and transportation of mordant fluids. For viscous fluids, Jaffrin and Sharpio [2] presented the earliest theoretical and experimental models for peristaltic transport. Many modern mechanical procedures have been investigated on the basis of peristaltic pumping for transporting fluids without internal moving parts (e.g., the blood pump in the heart, lung machines, and the peristaltic transport of noxious fluid in the nuclear industry). The combined effects of viscous dissipation and Joule heating on a magnetohydrodynamic Sisko nanofluid over a stretching cylinder were studied by Hussain et al. [3–5]. Hydromagnetic flow of a fluid in a uniform pipe with variable thickness was investigated by Hakeem et al. [6]. Peristaltic transport of a non-Newtonian fluid in an inclined channel was discussed by Vajravelu et al. [7]. Li et al. [8] discussed microstructure evolution and physical properties of laser-induced NbC-modified noncrystalline composites.

Study of peristaltic flow in the presence of magnetic field also has a lot of importance in daily life and engineering sciences. The effects of magnetohydrodynamics on peristaltic flows for different modes of heat transfer, such as conduction, convection, and radiation, are reported in Refs. [9,10]. For other studies regarding magnetohydrodynamic flows, see Ref. [11]. The rate of heat transfer is dependent on the temperatures of the systems and the properties of the prevailing medium through which the heat is transferred. Different authors have discussed the effect of force on heat convection and mass transfer [12,13]. Study of peristaltic flow in the presence of a magnetic field also has a lot of importance in daily life and engineering sciences. Stagnation electrical magnetohydrodynamic nanofluid mixed convection with a slip boundary on a stretching sheet was discussed in Ref. [14]. The effects of magnetohydrodynamics on peristaltic flows for different modes of heat transfer, such as conduction, convection, and radiation, are reported in Ref. [15]. Abbasi et al. [16,17] discussed magnetohydrodynamic mixed convective peristaltic motion of a nanofluid with Joule heating and thermophoresis effects.

Since the first investigation by Choi [18], the study of nanofluids has attracted the attention of many researchers because of their

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Nomenclature

	Velocity components (m/s)	T	Ratio of the heat capacity of the nanoparticle
$\overline{T}$	Temperature of fluid (K)	M	Hartman number (N s)/(C m)
$\overline{C}$	Nanoparticle concentration (mg/cm^3)	Δ	Wave number (dimensionless)
D_B	Brownian diffusion coefficient	C	Wave speed
M	Dynamic viscosity (N s/m^2)	B	Wave amplitude
		S_H	Schmidt number
		S_T	Soret number

tremendous applications in various fields of life, such as biomedical devices, treatment of tumors, nuclear reactors, microchips, cooling, radiators applications. Different authors have discussed heat transfer analysis for peristaltic flow of a Carreau-Yasuda fluid through a curved channel with a radial magnetic field [19,20]. Antolak-Dudka et al. [21] discussed algorithms for the estimation of transient surface heat flux during ultrafast surface cooling. Hsiao [22,23] discussed nanofluid flow with multimedia physical features for conjugate mixed convection and radiation.

In spite of all the aforementioned studies, boundary layer flow of a magnetohydrodynamic Sisko fluid with viscous dissipation and thermal conductivity has received little attention. Motivated by the previously cited literature and also by the various potential applications in engineering science and industry, we formulated a mathematical model for a non-Newtonian Sisko fluid in the presence of a magnetic field and heat transfer problem with the effects of both variable thermal conductivity and viscous dissipation. The governing equations for conservation of mass, momentum, and the heat equation have been simplified through the assumption of a low Reynolds number and a long wavelength. The physical features of pertinent parameters are discussed through graphs.

This article is structured as follows: In Section 2 we discuss the details of the mathematical formulation. Section 3 describes the proposed methods for solution of the governing partial differential equations. The analytical results are presented and discussed in Section 4. The conclusions are presented in Section 5.

2. Mathematical formulation

We analyze the peristaltic flow of an incompressible Sisko fluid in a uniform tube. The flow in the tube is a sinusoidal wave along the wall with constant speed c. The wall of the tube is defined as [24].

$$\overline{H} = a + b\sin\left[\frac{2\pi}{\lambda}(\overline{Z} - c\bar{t})\right], \tag{1}$$

where a is the radius of the tube at the inlet, b is the wave amplitude, λ is the wavelength, c is the propagation speed, and t is the time. The geometry of the problem is shown in Fig. 1.

The fundamental equations of continuity, momentum and nanoparticle concentration are

$$\frac{\partial\overline{U}}{\partial\overline{R}} + \frac{\overline{U}}{\overline{R}} + \frac{\partial\overline{W}}{\partial\overline{Z}} = 0, \tag{2}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}} + \overline{U}\frac{\partial}{\partial\overline{R}} + \overline{W}\frac{\partial}{\partial\overline{Z}}\right)\overline{U} = -\frac{\partial\overline{P}}{\partial\overline{R}} + \frac{1}{\overline{R}}\frac{\partial}{\partial\overline{R}}\left(\overline{R}\,\overline{S}_{\overline{R}\overline{R}}\right) + \frac{\partial}{\partial\overline{Z}}\left(\overline{S}_{\overline{R}\overline{Z}}\right) - \frac{\overline{S}_{\bar{\theta}\bar{\theta}}}{\overline{R}}, \tag{3}$$

$$\rho\left(\frac{\partial}{\partial\bar{t}} + \overline{U}\frac{\partial}{\partial\overline{R}} + \overline{W}\frac{\partial}{\partial\overline{Z}}\right)\overline{W} = -\frac{\partial\overline{P}}{\partial\overline{Z}} + \frac{1}{\overline{R}}\frac{\partial}{\partial\overline{R}}\left(\overline{R}\,\overline{S}_{\overline{R}\overline{Z}}\right) + \frac{\partial}{\partial\overline{Z}}\left(\overline{S}_{\overline{Z}\overline{Z}}\right) - \sigma B_0^2\overline{W}, \tag{4}$$

$$\rho c_p\left(\frac{\partial}{\partial\bar{t}} + \overline{U}\frac{\partial}{\partial\overline{R}} + \overline{W}\frac{\partial}{\partial\overline{Z}}\right)\overline{T} = \overline{S}_{\overline{R}\overline{R}}\frac{\partial\overline{U}}{\partial\overline{R}} + \overline{S}_{\overline{R}\overline{Z}}\frac{\partial\overline{W}}{\partial\overline{R}} + \overline{S}_{\overline{R}\overline{Z}}\frac{\partial\overline{U}}{\partial\overline{Z}} + \overline{S}_{\overline{Z}\overline{Z}}\frac{\partial\overline{U}}{\partial\overline{R}} + \overline{S}_{\bar{\theta}\bar{\theta}}\frac{\overline{U}}{\overline{R}} + K\left(\frac{\partial^2\overline{T}}{\partial\overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial\overline{T}}{\partial\overline{R}} + \frac{\partial^2\overline{T}}{\partial\overline{Z}^2}\right), \tag{5}$$

$$\left(\frac{\partial}{\partial\bar{t}} + \overline{U}\frac{\partial}{\partial\overline{R}} + \overline{W}\frac{\partial}{\partial\overline{Z}}\right)\overline{C} = D\left(\frac{\partial^2\overline{C}}{\partial\overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial\overline{C}}{\partial\overline{R}} + \frac{\partial^2\overline{C}}{\partial\overline{Z}^2}\right) + \frac{D\bar{k}_T}{\overline{T}_0}\left(\frac{\partial^2\overline{T}}{\partial\overline{R}^2} + \frac{1}{\overline{R}}\frac{\partial\overline{T}}{\partial\overline{R}} + \frac{\partial^2\overline{T}}{\partial\overline{Z}^2}\right). \tag{6}$$

The transformation relations between the two coordinate frames are

$$\bar{r} = \overline{R}, \quad \bar{z} = \overline{Z} - c\bar{t},$$

$$\bar{u} = \overline{U}, \quad \bar{w} = \overline{W} - c_1.$$

The corresponding boundary conditions are defined as

$$\frac{\partial\overline{W}}{\partial\overline{R}} = 0, \quad \frac{\partial\overline{T}}{\partial\overline{R}} = 0, \quad \frac{\partial\overline{C}}{\partial\overline{R}} = 0 \quad \text{at} \quad \overline{R} = 0,$$

$$\overline{W} = 0,\ K\frac{\partial\overline{T}}{\partial\overline{R}} = -\eta(\overline{T} - \overline{T}_0),\ \overline{C} = \overline{C}_0 \quad \text{at} \quad \overline{R} = \overline{H} = a + b\sin\left[\frac{2\pi}{\lambda}(\overline{Z} - c\bar{t})\right].$$

The constitutive equation for a Sisko fluid model is defined as

$$\bar{\mathbf{S}} = [a_1 + b_1(\sqrt{\Pi})^{n-1}]\overline{A_1},$$

$$\overline{A_1} = \overline{L} + \overline{L}^T,\ \overline{L} = \nabla\overline{V},\ \overline{\Pi} = \frac{1}{2}tr\overline{A_1}^2.$$

The nondimensional constants are as follows:

$$R = \frac{\overline{R}}{a}, \quad r = \frac{\bar{r}}{a}, \quad Z = \frac{\overline{Z}}{\lambda}, \quad z = \frac{\bar{z}}{\lambda}, \quad W = \frac{\overline{W}}{c}, \quad w = \frac{\bar{w}}{c},\ S = \frac{a\overline{S}}{c\mu_0},$$

$$U = \frac{\lambda\overline{U}}{ac}, \quad u = \frac{\lambda\bar{u}}{ac}, \quad t = \frac{c_1\bar{t}}{\lambda}, \quad \lambda_1 = \frac{\varepsilon_1 c_1}{a}, \quad \lambda_2 = \frac{\varepsilon_2 c_1}{a},$$

$$R_e = \frac{ac_1\rho}{\mu_0}, \quad \delta = \frac{a}{\lambda}, \quad h = \frac{\bar{h}}{a} = 1 + \varepsilon\sin(2\pi z), \quad P_r = \frac{\mu_0 c_p}{k},$$

$$E_c = \frac{c^2}{c_p T_0}, \quad \theta = \frac{\overline{T} - \overline{T}_0}{\overline{T}_0}, \quad \Pi = \frac{a\overline{\Pi}}{c}, \quad \alpha = \frac{k}{(\rho c)_f}, \quad p = \frac{a^2\bar{p}}{c_1\lambda\mu_0},$$

$$S_T = \frac{\rho Dk_T\overline{T}_0}{\mu T_0\overline{C}_0},\ S_H = \frac{\mu}{D\rho},\ \sigma = \frac{\overline{C} - \overline{C}_o}{\overline{C}_o}, \quad M = \sqrt{\frac{\sigma}{\mu}}B_0 a^2.$$

We suppose for a small Reynolds number $Re << 1$ and by the long-wavelength approximation $\delta << 1$, the flow inside the passage is very

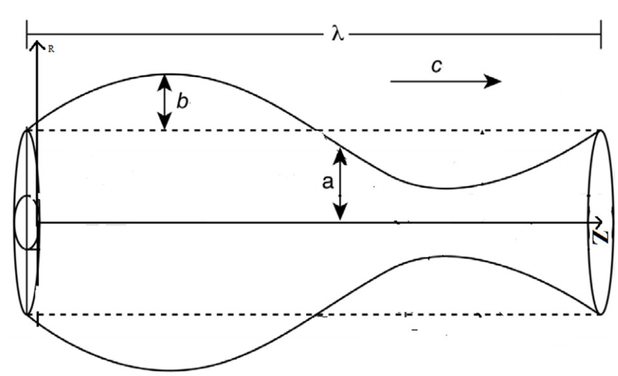

Fig. 1. A physical sketch of the problem.