Journal of Physics and Chemistry of Solids 76 (2015) 59–63

Contents lists available at ScienceDirect

Journal of Physics and Chemistry of Solids

journal homepage: www.elsevier.com/locate/jpcs

Mobility as controlled by the transverse component of phonon wave vector in quantum layers at low temperatures

S. Nag, D.P. Bhattacharya *

Department of Physics, Jadavpur University, Kolkata-700 032, India

ARTICLE INFO

Article history:
Received 26 May 2014
Received in revised form
14 July 2014
Accepted 17 August 2014
Available online 23 August 2014

Keywords:
A. Semiconductors
A. Surfaces
D. Phonons
D. Transport properties

ABSTRACT

Without resorting to either the Kawaji's simplified model of interaction with only two-dimensional phonons or to the equipartition approximation for the phonon distribution, the characteristics of the momentum relaxation time of the conduction electrons in a quantized surface layer for interaction with intravalley acoustic phonons have been analysed under the condition of low temperature. The scattering and the mobility characteristics thus obtained for an n-channel (1 0 0)-oriented Si inversion layer are apparently quite different from what follows in the traditional framework.

1. Introduction

There exists a large variety of semiconductor structures where the free electrons get confined in thin layers of the order of a few nm, thus forming a quasi two dimensional electron gas (Q2D). The electron gas there shows quantum size effects in its electrical properties and possesses a high value of zero-field mobility at low temperature. The study of the electrical transport properties of such systems is important in view of their involved physical characteristics which hold good promise for mesoscopic devices [1–3].

The electrical transport characteristic of any device structure is controlled by the dominant interactions of the electrons in the device material under the prevalent conditions. Some studies on the transport characteristics of the Q2D at low temperatures have already been made. Störmer et al. observed Shubnikov-de-Hass oscillations in a GaAs-Ga$_x$Al$_{1-x}$As heterostructures around 4.2 K and obtained mobility values at the low temperatures [4]. The Kawaguchi group experimentally determined the mobility characteristics in n-channel Si(1 0 0) inversion layers for temperatures below 70 K [5,6].

At the low temperatures, the impurity scattering may be dominant if the impurity content is sufficiently large. However, the interaction with the intravalley acoustic phonons is an intrinsic process and becomes dominant in the low temperature regime, $T_L \approx 20\ K$, for high purity materials [1–10].

For high temperatures, the phonon energy may be taken to be negligible in comparison to the average thermal energy of the carriers: $\hbar\omega_{\vec{q}} \ll k_B T_L$; $\hbar = h/2\pi$; h being Plank's constant, $\omega_{\vec{q}}\ (= u_l q)$, is the frequency of lattice vibration for any wave vector q, u_l being the acoustic velocity, and k_B is the Boltzmann constant. Under this condition, on making Taylor expansion of the exponential term in the Bose–Einstein distribution function and retaining up to the first order term $\left(\hbar\omega_{\vec{q}}/k_B T_L\right)$, the phonon distribution may be reduced to the simple equipartition law (eq.): $n_q + 1 \simeq n_q = \left(k_B T_L/\hbar\omega_{\vec{q}}\right)$. The theory of electron transport in Q2D, using the equipartition approximation for the distribution of acoustic phonons has been well developed [1,8,10,16–18]. However, at low temperatures, when the thermal energy becomes comparable with the phonon energy, the equipartition approximation is no longer valid. Lei, Birman and Ting have developed a non-Boltzmann theory of the steady state transport considering the full-phonon distribution at the low temperatures and obtained the transport characteristics in GaAs-Ga$_x$Al$_{1-x}$As heterojunctions [11]. In the similar framework Shinba et al. have developed the theory of phonon limited mobility in degenerate surface layers of Si(1 0 0) for temperatures around 50 K and have observed that the surfon scattering mechanism alone cannot explain the experimental observations [12].

Taking due account of the true phonon distribution (neq) by way of replacing the equipartition approximation by a suitable Laurent expansion, the theory of lattice controlled mobility in quantized layers at low temperatures have been developed by one of the present authors and another. The momentum relaxation time

* Corresponding author. Tel.: +91 33 2457 2612.
E-mail address: d_p_bhattacharya@rediffmail.com (D.P. Bhattacharya).

http://dx.doi.org/10.1016/j.jpcs.2014.08.006

and the mobility characteristics obtained for an n-channel (1 0 0)-oriented Si inversion layer appear to be significantly different from what the traditional theory predicts in the high temperature approximation [13].

The theoretical analyses of the electrical transport in Q2D are usually carried out in the light of Kawaji's simplified model of two dimensional phonons, (2dp), which assumes that even though the lattice wave is a three dimensional wave, the interaction of the quatised electrons will be confined to two dimensions [14]. As such, the phonon wave vector $\vec{q}$ that enters into the analyses are taken to be confined on the plane of the surface like the electron wave vector $\vec{k}$ [1,8,9,13–16]. But the phonon system is essentially three dimensional (3dp). The momentum conservation equation serves to restrict only the components of the phonon momentum in the direction parallel to the surface. Hence a more realistic theory demands that the transverse component of the phonon wave vector to be given due consideration [17–19].

The purpose of the present communication is to develop an analytical theory of the quasi-elastic scattering of the electrons due to interaction with intravalley acoustic phonons in a quantized surface layer under the condition of low temperature, without resorting either to the Kawaji's simplified model of two dimensional phonons or to the equipartition approximation for the phonon distribution. Ridley's momentum conservation approximation (MCA) [18,19] has been employed to account for the transverse component of the phonon wave vector. The dependence of the momentum relaxation time upon carrier energy, lattice temperature and the impurity concentration that determines the transverse dimension of the layer, is worked out. The analytical results which followed, are then used to calculate the dependence of the zero-field mobility on the lattice temperature and the impurity concentration of the layer. The numerical results thus obtained for the surface layers of Si are analysed and compared with other available results.

2. Development

Let us consider an ensemble of a non-degenerate Q2D formed over an oxide-semiconductor interface of surface of area A. The motion of the conduction electrons in the Q2D is like the classical free-electron motion on the x–y plane parallel to the interface, and the motion in the z-direction, perpendicular to the interface is quantised. The momentum relaxation time $\tau_{ac}\left(\varepsilon_{\vec{k}}\right)$ of an electron with energy $\varepsilon_{\vec{k}}$ due to elastic interaction with the intravalley acoustic phonons may be obtained from the perturbation theory, as [8,17]

$$\frac{1}{\tau_{ac}(\varepsilon_{\vec{k}})} \equiv P_{ac}(\varepsilon_{\vec{k}}) = \frac{2\pi}{\hbar}\sum_{\vec{Q}}\left|M(\vec{k},\vec{k}')\right|^2 \delta(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}}) \qquad (1)$$

where $\varepsilon_{\vec{k}} = (\hbar^2 k^2/2m_{\parallel}^*)$, is the kinetic energy of an electron with 2D wave vector $\vec{k}$ and $m_{\parallel}^*$ the effective mass of the electron parallel to the surface. The square of the matrix element for transition from the state $\left|\vec{k}\right\rangle$ to $\left|\vec{k}'\right\rangle$ due to interaction with intravalley acoustic phonons is given by

$$\left|M(\vec{k},\vec{k}')\right|^2 = \frac{E_1^2\hbar}{2Ad\omega_Q\rho_v}(q^2+q_z^2)|G(q_z)|^2\begin{bmatrix} N_{\vec{Q}} \\ N_{\vec{Q}}+1 \end{bmatrix}\delta_{k',k\pm q} \qquad (2)$$

here E_1 is the deformation potential constant, ρ_v is the mass density, $\omega_Q = u_l Q$, d is the width of the layer, q is the component of the three dimensional phonon (3dp) wave vector $\vec{Q}$ on the (x, y) plane and q_z is the transverse component along the z direction, $q^2+q_z^2 = Q^2$, $N_{\vec{Q}}$ is the phonon population, which cannot be described by the equipartition law under the condition of low temperature of interest here, $N_{\vec{Q}}$ or $(N_{\vec{Q}}+1)$ and the upper or the lower sign in (2) appear respectively for the processes of absorption (ab) and emission (em) of phonons, and $G(q_z)$ is the form factor.

As a result of quantum confinement of the electrons, the momentum is not conserved in all the three directions equally. As ensured by the Kronecker delta function, that appears in (2), the momentum conservation in the free propagation directions (on the x–y plane) is exact: $k' = k \pm q$. On the other hand, though it is favored, a strict conservation of momentum along the confining direction z, is not observed. The form factor $G(q_z)$ describes the degree of conservation along the z direction [18,19].

The dispersion relation for spherical constant energy surfaces is given by $\varepsilon_{\vec{k}} = (\hbar^2 k^2/2m_{\parallel}^*) + \varepsilon_n$, where ε_n is the energy eigenvalue of the nth subband [8].

Depending upon how the heterojunctions are manufactured, the quantum wells in which the 2DEG may exist, assume different forms. Such wells may be approximated as either square or triangular or parabolic [1,3]. For the silicon inversion layer however, the triangular well approximation is usually used. The analytical solution of the Schrodinger equation for such a potential well may be obtained in terms of Airy function, but the subsequent theoretical analysis of the layer characteristics appears to be cumbersome. Hence for the sake of convenience one can work with simpler wave functions like that proposed by Fang and Howard, which involves some variation parameter [1]. However, the problem under consideration here, proposes to account for the transverse component of the phonon wave vector in the light of the MCA. The application of MCA necessitates the evaluation of the form factor like $G(q_z)$ using the electron wave function that is consistent with the form of the well. Hence, in order to obtain manageable results we adopt the simple model of square well potential [18,19]. Thus for the electrons confined by an infinite well of width d, $\varepsilon_n = (\hbar^2\pi^2/2m_3^*)((n/d))^2$, where n is an integer and m_3^* is the effective mass perpendicular to the surface. Under the condition of low temperature and with low carrier densities, only the lowest subband $(n=1)$ is occupied and the higher subbands hardly play any significant role. Thus the layer thickness d is given by $[\hbar^2\pi^2 \in_{sc} \in_0 / 2m_3^* N_i e^2]^{1/3}$ where $\in_{sc}$ is the static dielectric constant of the semiconductor, $\in_0$ is the free space permittivity, N_i is the layer concentration and e is the electronic charge.

To a good approximation, the true $N_{\vec{Q}}$ at the low temperatures, may be represented by the Laurent expansion of the form [10]

$$N_{\vec{Q}}(X) = \sum_{m=0}^{\infty}\frac{B_m}{m!}X^{m-1}, \quad X \le \overline{X}$$
$$= \exp(-X), \quad X > \overline{X} \qquad (3)$$

where X is the normalized phonon wave vector given by $X = \hbar u_l Q / k_B T_L$; B_m are Bernoulli numbers. For all practical purposes $\overline{X}$ may be taken to be 3.5.

In the framework of MCA [18,19], with due regard to the transverse component q_z of the three dimensional phonon wave vector $\vec{Q}$, one can now carry out the summation in Eq. (1) for the condition of low temperature, without making the traditional equipartition approximation for the phonon distribution. Thus Eq. (1) can be expressed in the form

$$\frac{1}{\tau_{ac}(\varepsilon_{\vec{k}})} \equiv P_{ac}(\varepsilon_{\vec{k}}) = \frac{E_1^2}{8\pi^2\rho_v u_l}[J^{ab} + J^{em}] \qquad (4)$$

where J^{ab} and J^{em} are the integrals given by

$$J^{ab} = \iiint (q^2+q_z^2)^{1/2}\left|G(q_z)\right|^2 N_{\vec{Q}}\,\delta(\varepsilon_{\vec{k}'} - \varepsilon_{\vec{k}})\,q\,dq\,dq_z\,d\theta$$