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Formulation of the nonlinear sloshing-structure coupled problem based on the Hamiltonian mechanics for constraint systems

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ABSTRACT

This paper describes a formulation of a nonlinear sloshing problem based on the Hamiltonian mechanics. In particular, we focus on behavior of a liquid surface and a hydrodynamic force arising from the nonlinear sloshing in shallow water depth. It is well known that the water wave in shallow water depth shows the characteristic behaviors such as the solitary wave by inherent nonlinearities. Therefore, the effect of nonlinearity is significantly crucial for accurate predictions of the wave height and the hydrodynamic force. Although many researches have been studied for the feature of the nonlinear sloshing in shallow water depth, the theoretical analysis is essentially difficult because a lot of higher order nonlinear terms and sloshing modes have to be taken into account for accurate numerical predictions. Consequently, it yields complicated algebraic procedures. This study presents a formulation of nonlinear sloshing based on the canonical theory for constrained systems. In addition, the Dirichlet–Neumann operators developed by Craig and Sulem (1993) is introduced to obtain an asymptotic description for the kinematic boundary condition of the liquid surface. The proposed approach facilitates the consideration of the nonlinearity in the formulation. This study demonstrates analytical predictions considering up to the fourth-order nonlinear terms and higher-order sloshing modes and discusses adequate truncation orders for them. Moreover, experiments are conducted to measure time histories of the wave height and the nonlinear hydrodynamic force due to the sloshing in a rectangular tank subjected to a horizontal excitation. As the results of frequency analyses for the time histories, many frequency spectra with the integral multiples of the dominant frequency were observed. In particular, the only odd multiples of the dominant frequency were involved in the results of hydrodynamic force. These features were also obtained by the analytical predictions by the proposed method.

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1. Introduction

Behavior of a liquid surface in a partially filled liquid container is an important engineering problem for applications such as liquid storage tanks, liquid cargo transportations and a design of tuned liquid dampers (TLDs). Sloshing has been a subject of a great deal of literatures. A notable sloshing theory based on analytical asymptotic models with the modal expansion technique has been developed by Faltinsen and his colleagues. In a series of their papers, multidimensional modal approaches have been presented for the nonlinear sloshing with irrotational inviscid flows in a tank having a rigid wall. They developed general forms of multidimensional structures of equations for the wave elevation and the velocity potential in two-dimensional problems (Faltinsen et al., 2000). Moreover, the asymptotic models were improved by an alternative truncation strategy for approximations of nonlinear terms in a shallow water depth (Faltinsen and Timokha, 2001, 2002). These studies were further extended to a square-base tank (Faltinsen et al., 2003, 2005), a two-dimensional circular tank (Faltinsen and Timokha, 2010) and a spherical tank (Faltinsen and Timokha, 2013). In addition, a comprehensive review of theories and applications of sloshing were summarized by Ibrahim (2005). On the other hand, another asymptotic approach called the Dirichlent–Neumann operator (DNO) was proposed for the water wave problem by Craig and Sulem (1993), which was utilized as an approximation technique for a free surface kinematic boundary condition. This approach was employed to develop an efficient numerical simulation method for the nonlinear water wave problems by incorporating the fast Fourier transform technique. They also presented a theory based on the Hamiltonian mechanics with the DNO. The theory was applied to formulation of classical water wave problems such as the Boussinesq and the KdV equations (Craig and Groves, 1994) and a problem of internal waves between two fluids of different densities (Craig et al., 2005). In addition, we also introduced their method (Craig et al., 2005) into the sloshing problem in a tank, partially filled with two immiscible layered liquid, having an arbitrary cross-section shape and a flat bottom (Hara and Takahara, 2008a–c).

Development of accurate prediction methods for the nonlinear sloshing is also crucial for the design of tuned liquid dampers (TLDs). It can be expected that the performance of the TLD is significantly influenced by tuning natural frequencies of sloshing and structure, mass ratios, damping effects and inherent nonlinearities. In order to design efficient TLDs, behaviors of liquid inside the TLD tank have to be predicted precisely. In addition, TLD design parameters, such as the mass ratios and the damping ratios, have to be adjusted to optimal values. Although a lot of knowledge from tuned mass dampers based on the linear theory are applicable to designing the tuning of natural frequencies and the mass ratios, the damping effects and the inherent nonlinearities of sloshing motion make modeling and designing the TLD difficult. In general, inherent damping effects within the liquid inside the tank are less than an optimal design value as the TLD. Thus, additional damping devices, such as nets, slat screens and perforated thin plates are often equipped to increase dissipation energy. Kaneko and Ishikawa (1999) developed a numerical nonlinear model based on the shallow water theory. They investigated the effects of submerged nets on the TLD performance. Although the model has been assumed to be inviscid, incompressible and irrotational, they considered damping effects arising from the boundary layer along the wall and the free surface and the fluid drag on the submerged net. Moreover, Tait et al. (2005) evaluated a performance of the TLD equipped with multiple damping screens by introducing the similar model developed by Kaneko and Ishikawa (1999). Love and Tait (2010) have also developed a modal expansion approach with a modal damping accounting for energy losses arising from the fluid viscosity and the inclusion of damping screens. In the recent years, CFD techniques have been employed to calculate the sloshing motion in the TLDs. Maravani and Hamed (2011, 2014) have been discussed contributions of flow patterns due to the slat screens to the damping effects by in-house developed numerical algorithm. This study showed effects of the flow patterns due to the slat screen patterns on the performance of the TLD numerically (Marivani and Hamed, 2014). On the other hand, Love and Tait (2011, 2013) have investigated the inherent nonlinearities of the sloshing motion in a tank having irregular geometry with shallow water depth. Their model is based on the multimodal model developed by Faltinsen et al. (2000) and Faltinsen and Timokha (2001), it is applicable to analyses of tuned sloshing dampers having irregular tank geometries.

For designing of TLDs, the nonlinear hydrodynamic force in the shallow water depth is more effective than that in high water depth, that is, desirable performances as TLDs can be obtained by the nonlinear sloshing force in the shallow water depth rather than the high water depth. Therefore, knowledge of hydrodynamic force due to sloshing in the shallow water depth can contribute to designing of TLDs. However, Faltinsen et al. (2005) mentioned a difficulty of dealing with higher harmonics of a dominant frequency in predictions of hydrodynamic forces by the asymptotic approximation. They demonstrated comparisons of hydrodynamic forces calculated by simple formula with experimental results. The results implied that higher harmonics are crucial for the accurate evaluation of the hydrodynamic forces.

In this paper, the variational principle framework is employed to formulate the behavior of liquid surface in the tank partially filled with liquid. In particular, this study focuses on a formulation of the sloshing and the hydrodynamic force in the shallow water depth and its validations. The hydrodynamic force acting on the tank is evaluated by a fully coupled analysis of fluid–structure interaction problem. Since this system includes an intrinsic relation between the wave elevation and the translational displacement of tank, it requires a formulation as a constrained system. The main difficulty of this formulation is involved in this part. In order to resolve this problem, we try to introduce a formulation based on the canonical theory with the Poisson bracket formalism developed by Dirac (1964). This procedure enables us to formulate constraint systems systematically. Moreover, in order to describe the hydrodynamic force with higher order harmonics due to the sloshing in the shallow water depth, a restriction on the nonlinear terms in non-dominant modes is relaxed. In addition, the Dirichlet–Neumann operator (Craig and Sulem, 1993) is introduced to reduce the Laplace problem to the finite-dimensional system involving quantities evaluated at the liquid surface only. The equations of motion take account of