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Nonlinear aeroelastic modeling via conformal mapping and vortex method for a flat-plate airfoil in arbitrary motion

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A R T I C L E I N F O

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A B S T R A C T

A nonlinear aerodynamic modeling based on conformal mapping is presented to obtain semi-analytical formulas for the unsteady aerodynamic force and pitching moment on a flat-plate airfoil in arbitrary motion. The aerodynamic model accounts for large amplitudes and non-planar wake and is used to study the aeroelastic behavior of a flat-plate airfoil elastically connected to a support. The fluid is assumed to be inviscid and incompressible, while the flow is assumed to be attached, planar, and potential. Within these hypotheses, conformal mapping and a complex-potential representation of unsteady aerodynamics are used to simplify the theoretical formulation. The vorticity shed at the trailing edge is discretized in desingularized point vortices in order to allow free-wake dynamics. The unsteady aerodynamic model is validated with classical linearized formulations based on the assumption of small disturbances, and with experimental data and theoretical predictions for a large-amplitude pitch-up, hold, pitch-down maneuver. The aeroelastic model is then used to simulate the response of a flat-plate airfoil to sudden starts and body–vortex interactions. Numerical results show that the proposed approach can be an effective tool to model the aeroelastic behavior of an arbitrarily-moving wing section in a time-dependent potential stream of incompressible fluid.

1. Introduction

Aeroelastic design is typically tackled by means of linearized approaches and well-established associated computational tools. The linearization process typically includes the assumptions of linearized body kinematics and prescribed wake geometry. However, these simplifications may be not reasonable, for example, when analyzing aircraft configurations undergoing large-amplitude static and dynamic deflections. On the other hand, high-fidelity nonlinear aeroelastic models are computationally demanding, thus not suitable whenever multiple simulations have to be performed. Therefore, there is a need of simplified models, capable of accounting for relevant aerodynamic and structural nonlinearities with moderate computational burden, to be used in sensitivity analysis, optimization, and control.

In a linear framework, typical-section aeroelastic models were historically the first example of analytical tools used for prediction and design. The possibility to derive closed-form solutions for the unsteady aerodynamic loads under the assumption of small disturbances has made such models an important source of information on unsteady airfoil behavior

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Nomenclature

Latin symbols

$\boldsymbol{a}^{(k)}$	wake complex coefficient
$\boldsymbol{b}^{\mp}$	wake complex coefficient
$\boldsymbol{c}^{\mp}$	wake complex coefficient
$\boldsymbol{d}$	wake complex coefficient
D	drag
f_x	natural frequency of the horizontal linear spring
f_y	natural frequency of the vertical linear spring
f_α	natural frequency of the torsional spring
$\boldsymbol{F}^{(a)}$	aerodynamic force
$\boldsymbol{F}^{(e)}$	elastic force
$\boldsymbol{g}$	Fourier transform of the Green function
$\boldsymbol{g}_\varepsilon$	Fourier transform of the desingularized Green function
$\boldsymbol{G}$	Green function
$\boldsymbol{G}_\varepsilon$	desingularized Green function
$\boldsymbol{H}$	position of the flat-plate center
$\tilde{\boldsymbol{H}}$	position of the flat-plate center in aeroelastic equilibrium
$\boldsymbol{H}_e$	position of the flat-plate center in elastic equilibrium
$\boldsymbol{H}_0$	initial position of the flat-plate center
$\dot{\boldsymbol{H}}$	velocity of the flat-plate center
$\dot{\boldsymbol{H}}_0$	initial velocity of the flat-plate center
$\ddot{\boldsymbol{H}}$	acceleration of the flat-plate center
$\boldsymbol{i}$	imaginary unit
J_α	flat-plate moment of inertia with respect to the elastic center
k_x	constant of the horizontal linear spring
k_y	constant of the vertical linear spring
k_α	constant of the torsional spring
$\boldsymbol{K}$	Biot–Savart kernel
$\boldsymbol{K}_\varepsilon$	desingularized Biot–Savart kernel
l	flat-plate length
L	Lift
m	flat-plate mass
$M^{(a)}$	aerodynamic moment
$M^{(e)}$	elastic moment
$\boldsymbol{n}$	normal unit vector
p	pressure
R	circle radius
t	time
$\boldsymbol{u}$	local fluid velocity in the physical plane
$\boldsymbol{u}_b$	local body-boundary velocity in the physical plane
u_∞	modulus of the asymptotic flow velocity in the physical plane
$\boldsymbol{u}_\infty$	asymptotic flow velocity in the physical plane
$\boldsymbol{v}$	local fluid velocity in the transformed plane
$\boldsymbol{v}_\infty$	asymptotic flow velocity in the transformed plane
V_n	normal velocity of the flat-plate center
V_τ	tangent velocity of the flat-plate center
$\boldsymbol{w}$	complex potential in the physical plane
$\tilde{\boldsymbol{w}}$	complex potential in the transformed plane
$\boldsymbol{x}$	position in the physical plane
$\boldsymbol{x}_j$	position of the jth vortex in the physical plane
$\boldsymbol{x}_\pm$	position of the plate leading $(-)$ and trailing $(+)$ edge in the physical plane

Greek symbols

α	angle of attack
$\tilde{\alpha}$	angle of attack in aeroelastic equilibrium
α_e	angle of attack in elastic equilibrium
α_0	initial angle of attack
$\dot{\alpha}$	pitch rate
$\dot{\alpha}_0$	initial pitch rate
$\ddot{\alpha}$	pitch acceleration
β	phase of the asymptotic flow velocity in the physical plane
Γ_b	body circulation
$\tilde{\Gamma}_b$	body circulation in aeroelastic equilibrium
Γ_{b0}	initial body circulation
Γ_j	jth vortex circulation
$\Gamma^\star$	nascent vortex circulation
δ	parameter to locate the nascent vortex in the transformed plane
ε	parameter to desingularize the Biot–Savart kernel
$\boldsymbol{\zeta}$	position in the transformed plane
$\boldsymbol{\zeta}_j$	position of the jth vortex in the transformed plane
$\boldsymbol{\zeta}_\pm$	position of the plate leading $(-)$ and trailing $(+)$ edge in the transformed plane
$\boldsymbol{\zeta}^\star$	initial position of the nascent vortex in the transformed plane
μ	added-to-airfoil moment of inertia ratio
ρ	fluid density
σ	added-to-airfoil mass ratio
$\boldsymbol{\tau}$	tangent unit vector
φ	velocity potential
$\boldsymbol{\Phi}$	Schwarz function of the body boundary
χ	point on the unit circle
ψ	stream function
Ω_b	body cross-section

Operators

$\overline{(\cdot)}$	complex conjugate
$(\cdot)_n$	normal component
$(\cdot)_\tau$	tangent component
$(\cdot)_x$	real part
$(\cdot)_y$	imaginary part
$d(\cdot)$	differential
$\partial_t(\cdot)$	time derivative
$\partial_{\boldsymbol{x}}(\cdot)$	complex derivative in the physical plane
$\partial_\zeta(\cdot)$	complex derivative in the transformed plane