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Analytical formulation of 2-D aeroelastic model in weak ground effect

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ABSTRACT

This paper deals with the aeroelastic modeling and analysis of a 2-D oscillating airfoil in ground effect, elastically constrained by linear and torsional springs and immersed in an incompressible potential flow (typical section) at a finite distance from the ground. This work aims to extend Theodorsen theory, valid in an unbounded flow domain, to the case of weak ground effect, *i.e.*, for clearances above half the airfoil chord. The key point is the determination of the aerodynamic loads, first in the frequency domain and then in the time domain, accounting for their dependence on the ground distance. The method of images is exploited in order to comply with the impermeability condition on the ground. The new integral equation in the unknown vortex distribution along the chord and the wake is solved using asymptotic expansions in the perturbation parameter defined as the inverse of the non-dimensional ground clearance of the airfoil. The mathematical model describing the aeroelastic system is transformed from the frequency domain into the time domain and then in a pure differential form using a finite-state aerodynamic approximation (augmented states). The typical section, which the developed theory is applied to, is obtained as a reduced model of a wing box finite element representation, thus allowing comparison with the corresponding aeroelastic analysis carried out by a commercial solver based on a 3-D lifting surface aerodynamic model. Stability (flutter margins) and response of the airfoil both in frequency and time domains are then investigated. In particular, within the developed theory, the solution of the Wagner problem can be directly achieved confirming an asymptotic trend of the aerodynamic coefficients toward the steady-state conditions different from that relative to the unbounded domain case. The dependence of flutter speed and the frequency response functions on ground clearance is highlighted, showing the usefulness of this approach in efficiently and robustly accounting for the presence of the ground when unsteady analysis of elastic lifting surfaces in weak ground effect is required.

1. Introduction

Decreasing costs of air transportation without compromising safety levels is a fundamental objective in Aeronautics. This principle even had an impact on the concept of crafts designed to cruise close to the ground, like the well-known Russian ekranoplanes, presenting features similar not only to conventional aircrafts (they are airborne systems) but also to ships (they move close to the surface). In this perspective, flight in ground effect (GE) is one of the most critical conditions experienced by an airplane.

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The so-called ground effect for wings is described by Reeves (1993) as a ‘Phenomenon of aerodynamic, aeroelastic and aeroacoustic impacts on platforms wing in close proximity to an underlying surface’. Underlying the aerodynamic aspects, Rozhdestvensky (2006) identifies the GE[1] with its main consequence, that is, an increasing of aerodynamic efficiency E, commonly expressed as the ratio of the wing lift L to its drag D, *i.e.*, $E=L/D$. Thus, the enhanced efficiency can be obtained in two ways: by increasing lift (chord-wise GE), because of the higher pressure on the bottom of the wing section (see Abramowski, 2007), and by decreasing the (induced) drag (span-wise GE), due to the weaker development of vortices along the wing itself. Ground effect is commonly experienced by pilots during take-off or landing but have been also exploited as a permanent flight condition by wing-in-ground effect vehicles (briefly, WIGs), like the ekranoplanes mentioned above and other more recent flying prototypes (see, for instance, Matveev and Chaney, 2013).

In this paper, we will focus on the development of aeroelastic equations for 2-D wing sections under chord-wise ground effect, that is dominated by the ratio of the distance from the ground h_g to the airfoil chord c because of its 2-D nature. Another fundamental restriction concerns the range of values of h_g/c. Rozhdestvensky (2000) distinguishes between extreme (EGE) and moderate to weak ground effect (labeled simply as GE in this paper). The threshold between these two regimes is not a single value but rather an interval extending roughly from 1/4 to 1/2 for the ratio h_g/c, where approaches specifically developed for GE and EGE may coexist. By considering the GE regime, in this paper, this limit value will be further specified relatively to the developed theory.

The presence of the ground does not introduce any particular difficulty to Boundary Element Method (BEM) and Computational Fluid Dynamics (CFD) codes, especially when stationary loads are sought in moderate ground-effect regime. Potential flow solvers, in particular, exploit the method of images to ensure impermeability on the ground. Nonetheless, analytical solutions for 2-D potential flows around airfoils in ground effect are attractive because these intrinsically account for dependence on ground clearance. Plotkin and Kennel (1981) exploited asymptotic expansions of the involved variables with respect to the distance from the ground to achieve a cascade of equation where Sönghen inversion (see Bisplinghoff et al., 1996) could be performed to obtain the vortex intensity on the wing chord.

Though aeroelastic computations for wings in moderate to weak ground effect are easily affordable for potential flows, analytical models provide nonetheless useful reference solutions for the development of more complex codes. Moreover, they are faster to be employed as required for preliminary design and optimization procedures and some properties of the aeroelastic solution, *e.g.*, stability, are immediately available. For these reasons, this approach to fluid–structure interaction problems still keeps on receiving some attention from the scientific community in many cases (*e.g.*, see Berci et al., 2013). The governing equations can be written in a first-order form and this is also an advantage if nonlinearities are also included in the model so as to perform easily singular perturbation technique, or control has to be considered as well. Theodorsen (1935) theory of aeroelasticity of 2-D wing sections in an unbounded domain, if combined with the image method and with asymptotic expansions with respect to the h_g/c ratio, however, at first glance presents some difficulties. One of the critical points concerns the need for expanding in power series with respect to h_g/c the kernel of the circulatory term relative to the image airfoil, which is unsuitable to be inverted *a la* Sönghen. This occurs because polynomials diverge, whereas the contribution of the wake vorticity should damp out moving far from the trailing edge. Iosilevskii (2008) has carefully considered this critical aspect and has defined a limit position on the wake, depending on the ground clearance, beneath which such polynomial expansion still make sense. Beyond this limit, the wake contribution to the solution is found to be small, suitable to be evaluated numerically and essentially quadratic with respect to the inverse of the ground clearance. In this way, Iosilevskii could apply asymptotic expansions with respect to the inverse of the ground distance and, solving formally these equations at various orders, has provided the general analytical form of the unsteady aerodynamic load in the frequency domain for an airfoil with ground effect. To have a numerical comparison, he particularized the equations to study the variation of the aerodynamic lift coefficient of a heaving airfoil.

In this paper, starting from Iosilevskii formulation (terms up to second-order with respect to $(h_g/c)^{-1}$ are retained in the asymptotic solution of the lift coefficient, as shown in Section 2 and in Appendix A) the aeroelastic equations of motion of a 2-D oscillating airfoil in the presence of the ground, represented as a plane surface, has been issued to provide an extension of Theodorsen theory in weak GE.

First, the frequency domain formulation of the aerodynamic loads has been particularized for the case of a heaving and pitching airfoil (Section 2).

The expression of the circulatory terms in GE, involving Bessel functions in a more complicate way than in the case of Theodorsen theory, has suggested to provide simpler expression of these contributions through Padé approximation. Replacing transcendental functions with rational functions in the Laplace domain and so introducing a finite-state aerodynamics, the indicial function can be transformed back to the time domain and integro-differential (circulatory) terms can be represented via ordinary differential equations. Thus, the time-domain expression of the aerodynamic coefficients has been obtained, allowing us to study the so-called Wagner problem that accounts for the time-domain lift variation after a sudden start of an airfoil at a given incidence. In this way, full-coupled equations of an elastically constrained 2-D wing section are defined and the first-order differential form of these equations is finally provided (Section 3). On this basis, flutter analysis and response time histories are computed for a 2-D wing section.

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[1] This definition holds if weak to moderate ground effect is considered, see next for further specification.