Modern Electronic Materials 3 (2017) 40–49

Contents lists available at ScienceDirect

Modern Electronic Materials

journal homepage: www.elsevier.com/locate/moem

Quantum conductance staircase of holes in silicon nanosandwiches☆

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ARTICLE INFO

Keywords:
Quantum conductance staircase
Edge channels
Silicon nanosandwich
Odd and even fractional values

ABSTRACT

The results of studying the quantum conductance staircase of holes in one-dimensional channels obtained by the split-gate method inside silicon nanosandwiches that are the ultra-narrow quantum well confined by the delta barriers heavily doped with boron on the n-type Si (100) surface are reported. Since the silicon quantum wells studied are ultra-narrow (~2 nm) and confined by the delta barriers that consist of the negative-U dipole boron centers, the quantized conductance of one-dimensional channels is observed at relatively high temperatures ($T > 77$ K). Further, the current-voltage characteristic of the quantum conductance staircase is studied in relation to the kinetic energy of holes and their sheet density in the quantum wells. The results show that the quantum conductance staircase of holes in p-Si quantum wires is caused by independent contributions of the one-dimensional (1D) subbands of the heavy and light holes. In addition, the field-related inhibition of the quantum conductance staircase is demonstrated in the situation when the energy of the field-induced heating of the carriers become comparable to the energy gap between the 1D subbands. The use of the split-gate method made it possible to detect the effect of a drastic increase in the height of the quantum conductance steps when the kinetic energy of holes is increased; this effect is most profound for quantum wires of finite length, which are not described under conditions of a quantum point contact. In the concluding section of this paper we present the findings for the quantum conductance staircase of holes that is caused by the edge channels in the silicon nanosandwiches prepared within frameworks of the Hall geometry. This longitudinal quantum conductance staircase, G_{xx}, is revealed by the voltage applied to the Hall contacts, with the plateaus and steps that bring into correlation respectively with the odd and even fractional values.

1. Introduction

1.1. Conductance and conductivity

The scaling assumption is well-known to consider the relationship between the resistivity and the resistance of a hypercube:

$$R = \rho L^{(2-d)} \quad (1)$$

where L is the size of a hypercube and d denotes the dimension of a system [1,2].

Therefore the conductance, G, of a rectangular two-dimensional, d=2, conductor is directly proportional to its width, W, and inversely proportional to its length, L:

$$G = \sigma W/L \quad (2)$$

where the conductivity, σ, is a material parameter independent of the sample dimensions [3]. However contrary to the expectations that the conductance has to increase indefinitely when the length of a two-dimensional sample is reduced, the limiting value, G_c, has been found experimentally if its value becomes shorter than the mean free path ($L << L_m$) (Fig. 1a). Under these conditions corresponding to the ballistic transport regime, the resistance appears to result from the interface between the conductor and the large contact pads which are very dissimilar materials.

In order to provide the drop of the applied voltage entirely across the conductor, the contacts have to be more conducting than the conductor and exhibit the properties of reflectionless [3]. It should be noted that the reflection appears to be negligible with transmitting from the narrow conductor to the wide contact. Therefore the quasi-Fermi level F^+ for the $+k$ states is equal to μ_L even when a bias is applied (see Fig. 1b and c). No carriers originating in right contact ever makes its way to $+k$ state. Similar approach can be applied for the quasi-Fermi level F^- with $-k$ that is always equal to μ_R (see Fig. 1b and c). Thus, at low temperatures the current is equal to that carried by all the $+k$ states lying between μ_L and μ_R [3].

Hence the current appears to be calculated within frameworks of

☆Peer review under responsibility of the National University of Science and Technology MISiS.

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http://dx.doi.org/10.1016/j.moem.2017.04.003
Received 16 December 2016; Accepted 17 April 2017

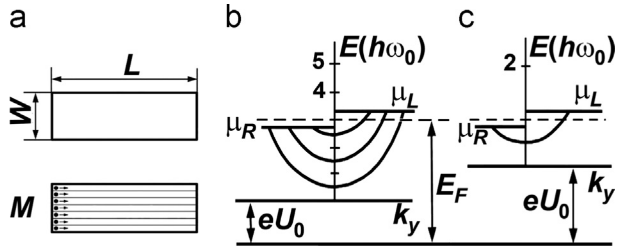

Fig. 1. (a) The narrow conductor that is able to demonstrate the ballistic transport effects when its length becomes shorter than the mean free path ($L << L_m$). The number of propagating modes (occupied one-dimensional subbands) depends on the relation between the width of the conductor and the Fermi wavelength, N=Int[$W/(\lambda_F/2)$], if the vertical transport is forbidden.

the model of the states in the narrow conductor that belong to different transverse modes or subbands [3]. Each mode has a dispersion relation $E(N,k)$ as sketched in Fig. 1a and b with a cut-off energy

$$\varepsilon_N = E(N, k = 0) \tag{3}$$

below which it cannot propagate. The number of transverse modes at an energy E is obtained by counting the number of modes having cut-off energies smaller than E:

$$M(E) = \sum_N \vartheta(E - \varepsilon_N) \tag{4}$$

The current carried by each transverse mode can be evaluated separately and add them up. In turn, a single transverse mode whose $+k$ states are occupied according to some function $f^+(E)$ is attributable to a uniform electron gas with n electrons per unit length moving with a velocity v carriers a current equal to env. Since the electron density associated with a single k-state in a conductor of length L is $(1/L)$ we can write the current I^+ carried by the $+k$ states as

$$I^+ = \frac{e}{L}\sum_k vf^+(E) = \frac{e}{L}\sum_k \frac{1}{\hbar}\frac{\partial E}{\partial k}f^+(E) \tag{5}$$

Assuming periodic boundary conditions and converting the sum over k into an integral according to the usual prescription

$$\sum_k \to 2(for\ spin) \times \frac{L}{2\pi}\int dk \tag{6}$$

The current is described by the following relationship

$$I^+ = \frac{2e}{h}\int_\varepsilon^\infty f^+(E)dE \tag{7}$$

where ε is the cut-off energy of the waveguide mode. This result can be extended to multi-moded waveguides:

$$I^+ = \frac{2e}{h}\int_\varepsilon^\infty f^+(E)M(E)dE \tag{8}$$

where the function $M(E)$ corresponds to the number of modes that are above cut-off at energy E. It should be noted that this result is generally independent of the actual dispersion relation $E(k)$ of the waveguide: the current carried per mode per unit energy by an occupied state is equal to $2|e|/h$ (which is about 80 nA/meV) [3].

1.2. Contact resistance

When the number of modes M is constant over the energy range $\mu_L > E > \mu_R$, the conductance of the narrow conductor depends linearly on the conductance of the single one-dimensional channel and the number of modes:

$$I = \frac{2e^2}{h}M\frac{(\mu_L - \mu_R)}{e} \Rightarrow G_C = \frac{2e^2}{h}M \tag{9}$$

Thus, the contact resistance which represents the resistance of a ballistic waveguide goes down inversely with the number of filled modes [4]:

$$G_C^{-1} \equiv \frac{(\mu_L - \mu_R)/e}{I} = \frac{h}{2e^2M} \approx \frac{12.9k\Omega}{M} \tag{10}$$

This relationship demonstrates that the contact resistance of a single-moded conductor, 12.9 kOhm, is certainly not negligible and can be measured if a single-moded ballistic conductor were sandwiched between two reflectionless conductive contacts. Besides, the contact resistance is very small if the wide conductors having thousands of modes are used.

To calculate the number of modes $M(E)$, the cut-off energies for the different modes ε_N are needed to be known. Since for wide conductors in zero magnetic field the precise nature of the confining potential is not important, the number of modes can be estimated simply by assuming periodic boundary conditions. The allowed values of k_y are then spaced by $2\pi/W$ (see Fig. 1a and b), with each value of k_y corresponding to a distinct transverse mode. At an energy E_f , a mode can propagate only if $-k_f < k_y < k_f$. Hence the number of propagating modes can be written as

$$M = Int\left[\frac{k_f W}{\pi}\right] = Int\left[\frac{W}{\lambda_f/2}\right] \tag{11}$$

where Int(x) represents the integer that is just smaller than x. Thus, the verification of the ballistic transport in the narrow conductor which is revealed by varying the number of filled channels results from the observation of the quantum conductance staircase. Firstly, this phenomenon has been found by measuring the quantum Hall resistance staircase as a function of the value of the strong magnetic field applied perpendicularly to two-dimensional semiconductor structures [1–3,5,6]. Secondly, progress in semiconductor nanotechnology made it possible to fabricate clean one-dimensional (1D) constrictions with low density of high-mobility charge carriers, which exhibit ballistic behavior if the mean free path is longer than the channel length [7–14]. Therefore, the conductance of such quantum wires prepared by the split-gate [7–13] and cleaved edge overgrowth [14] methods depends only on the transmission coefficient, T [4,15]:

$$G = g_s\frac{e^2}{h}M{\cdot}T \tag{12}$$

where M denotes the number of the highest occupied 1D subband, which is changed by varying the split-gate voltage, U_{sg} applied in the plane of two-dimensional semiconductor structure (Fig. 2b and c). Furthermore, the dependence $G(U_{sg})$ represents the quantum conductance staircase, because the conductance of a quantum wire is changed by the value of $g_s e^2/h$ each time when the Fermi level coincides with one of the 1D subbands [8,9]. Spin factor, g_s, describes the spin degeneration of the wire mode. The value of g_s is equal to two for non-interacting fermions if the external magnetic field is absent and becomes unity as a result of the Zeeman splitting of a quantum staircase in strong magnetic fields. The first step of the quantum conductance staircase has been found however to split into two parts even in the absence of external magnetic field [10–13]. The height of the substep that is dependent on temperature is usually observed to be about 0.7 of the first step value in a zero magnetic field. Two experimental observations indicate the importance of the spin component for the behavior of this $0.7 \cdot (2e^2/h)$ feature. First, the electron g - factor was found to increase from 0.4 to 1.3 as the number of occupied 1D subband decreases [10]. Second, the height of the $0.7 \cdot (2e^2/h)$ feature attains to a value of $0.5 \cdot (2e^2/h)$ with increasing external magnetic field [10–13]. These results have defined the spontaneous spin polarisation of a 1D gas in a zero magnetic field as one of possible mechanisms for the $0.7 \cdot (2e^2/h)$ feature [13,16–20] in spite of the theoretical prediction of a ferromagnetic state instability in ideal 1D systems in the absence of a magnetic field [21].

Studies of the quantum conductance staircase revealed by ballistic channels have shown that the $0.7 \cdot (2e^2/h)$ feature is observed not only in various types of the electron and hole GaAs based quantum wires [8–14,16–18,20,22], but also in the hole Si based quantum wires