Optics Communications 429 (2018) 144–151

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

An accurate phase shift extraction algorithm for phase shifting interferometry

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ARTICLE INFO

Keywords:
Interferometry
Interference fringes
Phase-shift extraction
Ellipse fitting

ABSTRACT

The accuracy of phase shift extraction has a significant influence on measurement results in surface micro-topography interferometry. Phase shifting errors are mainly caused by nonlinearity of the employed phase shifter, environmental turbulence, camera imperfection and so on. In this paper, a general algorithm based on Lissajous figures and ellipse fitting is proposed for extracting the phase distribution from a set of phase-shifting interferograms with random noise. Two sets of pixels with $\pi/2$ phase difference in all the investigated interferograms are selected and used for ellipse fitting. Both numerical simulations and optical experiments have proven the validity, rapidity, and accuracy of the proposed method. Experiments show that the proposed method is a general phase extraction method, which can work for straight fringe patterns, circle fringes patterns and other anomalous features.

1. Introduction

High precision surface topography measurement has many applications in areas such as integrated circuits and MEMS [1–5]. Phase-shifting interferometry (PSI) is one of the most widely used techniques in surface topography measurement as it is non-contact, non-destructive and has high accuracy [6–10]. To conduct a complete phase-shift measurement, a piezoelectric ceramic transformer (PZT) is generally used as the phase shifter. However, due to the hysteresis nonlinearity of the PZT and the instability of the environment (including ambient vibration and air shock), random sampling errors are unavoidable in phase shift measurement. The accurate extraction of phase shifts is a challenging task and has significant influence on measurement results.

To reduce the errors in the phase-shift extraction, many researchers have tried changing the configuration of interferometers or improving phase-shifting algorithms. For example, simultaneously phase shifting interferometry (SPSI) can effectively avoid environmental vibrations by collecting interferograms instantaneously [11,12], but the limitation of this method is the complicated hardware configuration. In fact, there has been more research focusing on modification of phase-shifting extraction algorithms. All the existing algorithms can be divided into two classes: iterative algorithms [13–15] and non-iterative algorithms [16–18].

In 1982, Morgan developed a least-squares iterative algorithm that estimates phases and their perturbation caused by linear time-dependent drifts [19]. In 1991, Okada et al. proposed a least-square-based iterative algorithm to solve a set of approximate linear equations iteratively, which allows the phase shift amounts and phase distributions to be determined simultaneously [20]. In 2007, Wang et al. proposed an advanced iterative algorithm (AIA) which can extract both initial phase distribution and phase shift amounts using three randomly shifted interferograms [21], overcoming the limitation found in conventional iterative algorithms that the number of frames must be at least four. In simulation, the phase extraction errors of the AIA algorithm with three frames are less than 0.0152 rad [15,21]. In 2010, Q. Kemao et al. proposed a windowed Fourier ridges and least squares fitting (WFRLSF) [22], and presented the phase shift errors of the algorithms: the AIA, the WFRLSF, the windowed Fourier transform (WFF) + AIA + WFF, and the WFF + WFRLSF + WFF. In simulation the phase shift errors are less than 0.06 rad, and as noise level is increased the phase shift error ranged from 0.026 to 0.29 rad. Recently, R. Zhu et al. extracted measurement phases from two phase-shifting fringe patterns using the spatial-temporal fringes method [23]. The surface error using this method is $1.8 \times 10^{-3}\lambda$.

Compared with the iterative algorithms above, the approaches based on non-iterative solutions are intended to find the optimal results in less

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https://doi.org/10.1016/j.optcom.2018.08.005
Received 26 March 2018; Received in revised form 26 July 2018; Accepted 4 August 2018
Available online 7 August 2018

time. Lissajous ellipse fitting, an example of a non-iterative approach, has proved to be an outstanding algorithm to extract phase shifts with high accuracy and efficiency [24]. In 1994, Farrell and Player used a pair of different pixels in fringe field (inter-pixel) to create Lissajous figures, from which they then calculated phase shift amounts, intensity bias and intensity modulation at each pixel using ellipse fitting, under the condition of both unequal and unknown phase steps [25]. The experimental result of wavefront reconstruction shows that the accuracy of the proposed algorithm is 0.032 rad. In addition, they mentioned that if the phase difference between the pixel pairs is close to $\pm n\pi$, Bookstein's algorithm would fail. In 2015, Fengwei Liu et al. proposed to correct the dynamic random phase shift errors by transforming the Lissajous ellipse to a unit circle (ETC) [26]. They deduced that the phase extraction error can be compensated with the unit circle and that the accuracy of correcting the phase extraction error is mainly dependent on the parameters of the ellipse. Experimental results show that the ETC method has similar precision comparable to AIA. Both Refs. [25] and [26] mentioned that when the phase difference between the pair of pixels equals $\pi/2$ the extracted phase will be most accurate. However, a way of choosing the pixel pairs is lacking. Moreover, the accuracy and reliability of the phase extraction will be appreciably affected by random phase shifting noise if only a single pair of pixels is used for Lissajous ellipse generation.

By using a series of pixel pairs with $\pi/2$ phase difference, we introduce a high-precision phase extraction method based on a least-squares algorithm and general ellipse fitting. The proposed method chooses the same region from all interferograms, and the average intensity of each selected region is calculated. Then two index numbers of interferograms sequence are obtained by seeking two points with $\pi/2$ phase difference within the average intensity array. Following this, two groups of pixels with $\pi/2$ phase difference in all the investigated interferograms are selected and used for ellipse fitting. The proposed method is not restricted in 3-interferograms, and it can effectively suppress random phase-shifting errors.

2. Theoretical analysis

2.1. Seeking two interferograms with $\pi/2$ phase difference

The phase-shift fringe pattern generated by phase-shifting can be expressed as

$$I_i(x,y) = A_i(x,y) + B_i(x,y)\cos[\varphi(x,y) + \theta_i] + N_i(x,y) \tag{1}$$

where i denotes the sequence number of phase-shifting interferogram, (x,y) represents the coordinate of an arbitrary pixel, $I_i(x,y)$ is the intensity at pixel location (x,y), A_i and B_i represent the background intensity and the modulation amplitude respectively, $\varphi(x,y)$ is the initial phase, $N_i(x,y)$ denotes the random noise, and θ_i describes the phase shift of the ith interferogram. θ_i is the main phase shift parameter that we need to obtain in each interference sequence.

With ellipse fitting, the accuracy of extracted phase shift θ_i can be improved by using two sets of signals that are reliable with phase difference $\pi/2$ [25]. The set of points can be selected in two interferograms that have a feature with $\pi/2$ phase difference. Now we explain how to find these two interference patterns.

Firstly, a center area (here, a rectangle with a length of $a_2 - a_1$ and a width of $b_2 - b_1$ as an example) of each interferogram is chosen to calculate the average intensity of all pixels in accordance with the expression.

$$G_i = \frac{1}{(a_2-a_1)*(b_2-b_1)} \sum_{k=a_1}^{a_2} \sum_{l=b_1}^{b_2} I_i(x_k, y_l) \tag{2}$$

where G_i represents the average gray value of all the pixels in the selected region. The average value G_i follows a sine curve, as show in Fig. 1, in which the maximum and minimum values of G_i, *i.e.*, G_{max} and G_{min}, are marked with red points.

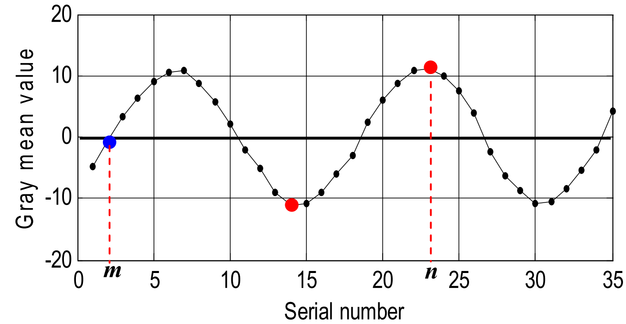

Fig. 1. The tendency of intensity curves with 35 fringes. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

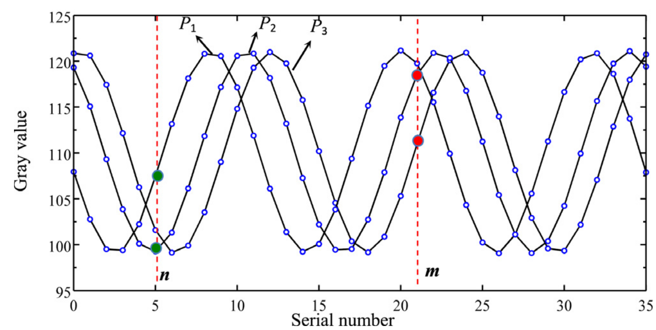

Fig. 2. Select the pixels with the same gray-scale change tendency. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

We seek two values in the array G_i with $\pi/2$ phase difference. According to the law that the phase difference between the maximum value and the middle value (amplitude = 0) is $k\pi + \pi/2$, we can obtain the sequence numbers m and n from the following equation:

$$\begin{cases} m = i & if \left| G_i - \dfrac{G_{max} + G_{min}}{2} \right| = Min \\ n = i & if\ G_i = G_{max} \end{cases} \tag{3}$$

The phase difference between the mth and the nth interferograms is close to $k\pi + \pi/2 + \varepsilon$, where ε is a deviation close to zero. The corresponding intensities I_m and I_n can be expressed as:

$$\begin{cases} I_m(x,y) = A(x,y) + B(x,y)\cos[\varphi(x,y) + \theta_m] \\ I_n(x,y) = A(x,y) + B(x,y)\cos[\varphi(x,y) + \theta_m + k\pi + \pi/2 + \varepsilon] \end{cases} \tag{4}$$

where θ_m is the mth phase shift value.

2.2. Seeking the pixels with same phase change tendency in mth and nth interferograms

Due to the unavoidable random noise, the Lissajous fitting accuracy will be unreliable if just a single pixel is used in each interferogram. Therefore, we propose to replace a single pixel by the average value of a group of pixels selected from the interferograms I_m *and* I_n in a certain way. Specifically, we take all pixels with the same grayscale change tendency within a certain range, such as ($-\pi$ < phase < 0). For example, in Fig. 2, P_1, P_2 and P_3 represent the intensity distributions of three points ((x_{p1}, y_{p1}), (x_{p2}, y_{p2}), (x_{p3}, y_{p3})) in different interferograms. The red pixels ((x_{p2}, y_{p2}), (x_{p3}, y_{p3})) in the mth interferogram will be selected for Lissajous fitting because at each of these points the intensity is increasing, however the pixel (x_{p1}, y_{p1}) is not selected as its intensity is decreasing. Similarly for the nth interferogram, suitable pixels have been marked in green in Fig. 2.