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Polarisation-dependent transformation of vortex beams when focused perpendicular to the crystal axis

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ABSTRACT

The polarisation-dependent effect of the astigmatic transformation of vortex beams focused perpendicular to the axis of a uniaxial crystal is investigated theoretically and experimentally. The dependence of the transformation on the polarisation state of a beam incident on a crystal was studied numerically in detail, showing the influence of the polarisation state on the individual electric field components of ordinary and extraordinary beams. The experimental results are in agreement with theoretical studies. The results obtained can be used for the generation and detection of optical vortex beams in optical communication systems and in the field of polarisation-controllable optical manipulation of nano- and microparticles.

0. Introduction

The polarisation-phase state of a laser beam plays a significant role in sharp focusing [1–7] and in the laser radiation-matter interaction [8–13]. At the same time, optical devices that make it possible to transform the polarisation properties of electromagnetic radiation are of increasing interest for practical applications [14–20]. One of the compact and wavelength-independent instruments used for polarisation transformations is the anisotropic crystal [21–25]. The propagation of laser modes with a high numerical aperture in a medium with strong anisotropy leads to complex polarisation-phase transformations. The transformation of vortex laser beams is of special interest. Most of the existing works have been devoted to the study of the propagation of singular beams along the crystal axis [24–30] or under a slight slope to the crystal axis [31–33]. In particular, when a circularly polarised beam propagates along the crystal axis, the spin angular momentum is transformed into an orbital angular momentum (OAM). The propagation of various types of laser beams perpendicular to the crystal axis has been investigated [23,31–39], with a notable astigmatic distortion of the ring structure of the beam observed for Bessel beams [23,36,39].

In this paper, the effect of the astigmatic transformation of vortex Gaussian beams focused perpendicular to the axis of an anisotropic crystal is studied theoretically, numerically and experimentally for different polarisation states of the incident beam. For Gaussian beams, the astigmatic transformation is less noticeable, because natural crystals have, as a rule, a small relative difference between the ordinary and extraordinary refractive indices. Sharp focusing of vortex Gaussian beams is used to intensify the astigmatic effect and make it visually evident. In this case, the influence of the polarisation state of the illuminating beam becomes particularly important. We show the polarisation dependence of the astigmatic transformation and investigate in detail the influence of the initial polarisation state on the individual components of the electric field of ordinary and extraordinary beams both theoretically and numerically.

Polarisation-sensitive dynamic control of the structure of the generated light field distributions can be used in the field of optical manipulation of nano- and microparticles in liquid and gaseous media [40–42]. In addition, the possibility of crystals performing the astigmatic transformation allows for the construction of compact devices for detection of OAM in quantum and optical communication systems working on the principle of OAM division multiplexing [43–45]. The results obtained demonstrate the possibility of efficiently detecting optical vortex beams with a topological charge up to 20. Moreover, the experimental results are in complete agreement with theoretical studies.

1. Theoretical analysis

The diffraction patterns of laser beams propagating along or perpendicular to the axis of the crystal differ significantly. As a rule, in the case of nonparaxial propagation or focusing of beams along the crystal axis, polarisation-phase transformations of the beams occur [24–30]. When beams pass perpendicular to the crystal axis, astigmatic transformations are observed, and these are especially noticeable for

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Bessel beams [23,36,39]. To make these transformations significant for Gaussian beams, it is necessary to use focusing [25,33,38,46]. In this case, the Gaussian beams become non-paraxial, and the influence of the polarisation of the illuminating beam becomes particularly important.

Let us analyse the nonparaxial propagation of an electromagnetic wave perpendicular to the crystal axis using the Rayleigh–Sommerfeld vector integrals obtained for an anisotropic medium [47]. Let the axis of the crystal be directed along the coordinate axis Y, and the dielectric permittivity tensor then has the following form (in the absence of charges, the magnetic permeability is assumed to be equal to $\mu = 1$ everywhere):

$$\vec{\varepsilon}_y = \begin{pmatrix} \varepsilon_0 & 0 & 0 \\ 0 & \varepsilon_1 & 0 \\ 0 & 0 & \varepsilon_0 \end{pmatrix}, \tag{1}$$

where ε_0 and ε_1 are the ordinary and extraordinary dielectric permittivities of a uniaxial crystal, respectively.

The field for the electrical vector of the electromagnetic wave propagating along the optical axis z has the following form [47]:

$$\begin{aligned} \mathbf{E}(u,v,z) &= \begin{pmatrix} E_x(u,v,z) \\ E_y(u,v,z) \\ E_z(u,v,z) \end{pmatrix} \\ &= \frac{2\pi}{\lambda^2}\iint \left\{ \frac{z\sqrt{\varepsilon_0}}{R_0^2} \mathbf{W}_0\left(\alpha_{c0},\beta_{c0}\right)\mathbf{E}_\perp(x,y,0)\exp\left\{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_0}R_0\right\}\mathrm{d}x\mathrm{d}y \right\} \\ &+ \frac{2\pi}{\lambda^2}\iint \left\{ \frac{z\varepsilon_1}{\sqrt{\varepsilon_0}R_1^2} \mathbf{W}_1\left(\alpha_{c1},\beta_{c1}\right)\mathbf{E}_\perp(x,y,0)\exp\left\{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_0}R_1\right\}\mathrm{d}x\mathrm{d}y \right\}, \end{aligned} \tag{2}$$

where $\mathbf{E}_\perp(x,y,0) = \begin{pmatrix} E_x(x,y,0) \\ E_y(x,y,0) \end{pmatrix}$ are transverse components of the input field,

$$\begin{cases} \alpha_{c0} = \dfrac{\sqrt{\varepsilon_0}(u-x)}{R_0} \\ \beta_{c0} = \dfrac{\sqrt{\varepsilon_0}(v-y)}{R_0} \end{cases}, \quad \begin{cases} \alpha_{c1} = \dfrac{\varepsilon_1(u-x)}{\sqrt{\varepsilon_0}R_1} \\ \beta_{c1} = \dfrac{\sqrt{\varepsilon_0}(v-y)}{R_1} \end{cases}, \tag{3}$$

$R_0 = \sqrt{(u-x)^2+(v-y)^2+z^2}$ and $R_1 = \sqrt{\frac{\varepsilon_1}{\varepsilon_0}(u-x)^2+(v-y)^2+\frac{\varepsilon_1}{\varepsilon_0}z^2}$.

In Eq. (2), the first integral corresponds to the ordinary beam, and the second corresponds to the extraordinary beam. The corresponding polarisation transformations are described by the following matrices:

$$\mathbf{W}_0(\alpha,\beta) = \begin{pmatrix} 1 & \dfrac{\alpha\beta}{\varepsilon_0-\beta^2} \\ 0 & 0 \\ -\dfrac{\alpha}{\sqrt{\varepsilon_0-\alpha^2-\beta^2}} & -\dfrac{\alpha^2\beta}{\sqrt{\varepsilon_0-\alpha^2-\beta^2}\left(\varepsilon_0-\beta^2\right)} \end{pmatrix}, \tag{4}$$

$$\mathbf{W}_1(\alpha,\beta) = \begin{pmatrix} 0 & -\dfrac{\alpha\beta}{\varepsilon_0-\beta^2} \\ 0 & 1 \\ 0 & -\dfrac{\beta\sqrt{\varepsilon_1-\alpha^2-\frac{\varepsilon_1}{\varepsilon_0}\beta^2}}{\varepsilon_0-\beta^2} \end{pmatrix}. \tag{5}$$

As follows from Eqs. (2)–(5), the total distribution essentially depends on the polarisation of the incident beam. In particular, the y-component will be present only in the extraordinary beam in the case of the presence of energy in this component in the input field $\mathbf{E}_\perp(x,y,0)$. If the field is initially x-polarised, then the extraordinary beam will be absent. Let the input field be defined as follows:

$$\mathbf{E}_\perp(x,y,0) = E_0(x,y)\begin{pmatrix} c_x \\ c_y \end{pmatrix}, \tag{6}$$

where $\mathbf{c} = (c_x, c_y)^T$ is the input polarisation vector.

First, considering the x-linearly polarised field $\mathbf{c} = (1,0)^T$, the field in the crystal will consist only of the ordinary beam, and Eq. (2) will take the following form:

$$\mathbf{E}^{x-lin}(u,v,z) = \frac{2\pi}{\lambda^2}\iint \frac{z\sqrt{\varepsilon_0}}{R_0^2}E_0(x,y)\begin{pmatrix} 1 \\ 0 \\ -\dfrac{\alpha_{c0}}{\sqrt{\varepsilon_0-\alpha_{c0}^2-\beta_{c0}^2}} \end{pmatrix} \times\exp\left\{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_0}R_0\right\}\mathrm{d}x\,\mathrm{d}y. \tag{7}$$

Thus, the field will remain basically x-linearly polarised, although some of the energy will go to the longitudinal component. Since this part is insignificant even with sharp focusing [30,46], only the transverse components will be considered. In particular, instead of Eq. (7), one can write the following:

$$E_x^{x-lin}(u,v,z) = \frac{2\pi z\sqrt{\varepsilon_0}}{\lambda^2}\iint \frac{E_0(x,y)}{(u-x)^2+(v-y)^2+z^2} \times\exp\left\{i\frac{2\pi\sqrt{\varepsilon_0}}{\lambda}\sqrt{(u-x)^2+(v-y)^2+z^2}\right\}\mathrm{d}x\,\mathrm{d}y. \tag{8}$$

The field defined by Eq. (8) is the Rayleigh–Sommerfeld scalar integral [48], describing the propagation of the field in a homogeneous medium with a refractive index $n_0 = \sqrt{\varepsilon_0}$.

If the input field is y-linearly polarised $\mathbf{c} = (0,1)^T$, then even without taking into account the longitudinal component, the field will be more complicated:

$$\begin{aligned} \mathbf{E}_\perp^{y-lin}(u,v,z) &= \frac{2\pi}{\lambda^2}\iint\left\{\frac{z\sqrt{\varepsilon_0}}{R_0^2}E_0(x,y)\begin{pmatrix} \dfrac{\alpha_{c0}\beta_{c0}}{\varepsilon_0-\beta_{c0}^2} \\ 0 \end{pmatrix} \times\exp\left\{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_0}R_0\right\}\mathrm{d}x\mathrm{d}y\right\} \\ &+ \frac{2\pi}{\lambda^2}\iint\left\{\frac{z\varepsilon_1}{\sqrt{\varepsilon_0}R_1^2}E_0(x,y)\begin{pmatrix} -\dfrac{\alpha_{c1}\beta_{c1}}{\varepsilon_0-\beta_{c1}^2} \\ 1 \end{pmatrix}\exp\left\{i\frac{2\pi}{\lambda}\sqrt{\varepsilon_0}R_1\right\}\mathrm{d}x\mathrm{d}y\right\}. \end{aligned} \tag{9}$$

From Eq. (9), the x-component will be a destructive superposition of the ordinary and extraordinary beams, and the y-component will contain only the extraordinary beam. The latter is astigmatic distorted according to the core of the transformation

$$R_1 = \sqrt{\frac{\varepsilon_1}{\varepsilon_0}(u-x)^2+(v-y)^2+\frac{\varepsilon_1}{\varepsilon_0}z^2}.$$

Note that in an isotropic medium, when the critical points (3) are equal to each other ($\alpha_{c1} = \alpha_{c0}$, $\beta_{c1} = \beta_{c0}$), the x-component will disappear and Eq. (9) will reduce to the scalar Rayleigh–Sommerfeld integral. For an anisotropic crystal $\alpha_{c1} \neq \alpha_{c0}$, $\beta_{c1} \neq \beta_{c0}$, there will be some (small) energy in the component perpendicular to the axis of the crystal.

If the initial polarisation has energy in both transverse components (for example, circular or elliptical polarisation), then a beam propagating perpendicular to the crystal axis will have an undistorted distribution (as in an isotropic medium) in the component perpendicular to the crystal axis, and an astigmatically transformed distribution will be observed in the component parallel to the crystal axis. Thus, by changing the polarisation at the input and using a polarisation analyser, we can dynamically control the field intensity distribution at the crystal output. A similar effect can be obtained for a linearly polarised beam by rotating the crystal axis around the optical axis.

2. Numerical simulation

A numerical study of focused vortex Gaussian beam propagation perpendicular to the axis of the calcite crystal ($CaCO_3$) having permittivities $\varepsilon_0 = 2.232$ and $\varepsilon_1 = 2.376$ was performed. To compare vortex Gaussian