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Phase transition of cosh-Airy beams in inhomogeneous media

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ABSTRACT

The cosh-Airy beam is a kind of nondiffracting beam that can be considered as a superposition of two Airy beams with different decay factors and the same propagation trajectory. In this study, we investigate the propagation of a cosh-Airy beam in a quadratic-index inhomogeneous medium and show the periodic phase transition of the cosh-Airy beam, which exhibits a beam transition from the Airy profile to a cosine Gaussian profile periodically. After the phase transition, the field distribution of the cosh-Airy beam is inverted, but the self-acceleration direction of the cosh-Airy beam is unchanged. We show that because of the phase difference of two components of the cosh-Airy beam, the interference effect of the two components plays an important role in the phase transition of the cosh-Airy beam in inhomogeneous medium. Our study may lead to some new potential applications in the manipulation of the Airy beam, signal processing, and other fields.

1. Introduction

In recent years, Airy beams have attracted immense attention owing to their fascinating propagation properties [1,2]; e.g., they are non-diffracting, self-healing [3,4], and self-accelerating. Owing to these interesting propagation properties, Airy beams have inspired a variety of applications, including optical micro-manipulation [5]; plasma channel generation [6]; plasmonic energy routing [7]; light bullets [8]; and the generation of electron Airy beams [9], surface Airy waves [7], acoustic Airy beams [10], water-wave Airy pulses [11], and electron plasma Airy waves [12].

The propagation of Airy beams in various optical media is an important issue in optics. Researchers have investigated the propagation of Airy beams in non-linear media [13–17], chiral media [18,19], nematic liquid crystals [20], uniaxial crystals [21,22], photonic crystals [23], and inhomogeneous media [24–29]. Studies shown that the non-linearity or inhomogeneity of the medium can change the self-acceleration [24,25], self-focusing (or defocusing) [27], or polarization-dependent rotation of Airy beams owing to the spin–orbit coupling [29].

Cosh-Airy beams have similar propagation properties to Airy beams [30]. They possess more manipulation degrees of freedom than Airy beams and can be considered as a superposition of two Airy beams with the same propagation trajectory and different decay factors. In Ref. [27], Zhang etc. show a phase transition of Airy beams in a parabolic potential, which indicates that the beam profile changes from the Airy profile to the Gaussian profile periodically. The following question then arise. Is the propagation of the cosh-Airy beam the same as that of the Airy beam? There is inevitably a phase difference between the two components of the cosh-Airy beam; how does this phase difference affect the propagation of the cosh-Airy beam in an inhomogeneous medium?

In this study, we investigate the evolution of cosh-Airy beams in a quadratic-index inhomogeneous medium. We show that the periodic self-acceleration and field distribution inversion of the cosh-Airy beam, and know that the difference between the two inversion locations is a quarter of the evolution period of the beam. Moreover, the cosh Airy beam undergoes a phase transition, exhibiting a beam profile change from the Airy profile to the cosine Gaussian profile periodically. This differs from the case of the Airy beam. The generation of the cosine Gaussian profile is attributed to the interference of the two components of the cosh-Airy beam.

The organization of this paper is as follows. In Section 2, we present a theoretical model of the propagation of cosh-Airy beams in the quadratic-index inhomogeneous medium. In Section 3, we give the numerical calculation results for the propagation of cosh-Airy beams in the quadratic-index medium. The paper is concluded in Section 4.

2. Theoretical model

The paraxial propagation of a cosh-Airy beam can be determined using the Huygens–Fresnel integral, as follows [31]

$$u(x,z) = \sqrt{\frac{k}{2\pi i B}} \int u(x_1, z=0) \exp\left\{\frac{ik}{2B}(Ax_1^2 - 2xx_1 + Dx^2)\right\} dx_1, \tag{1}$$

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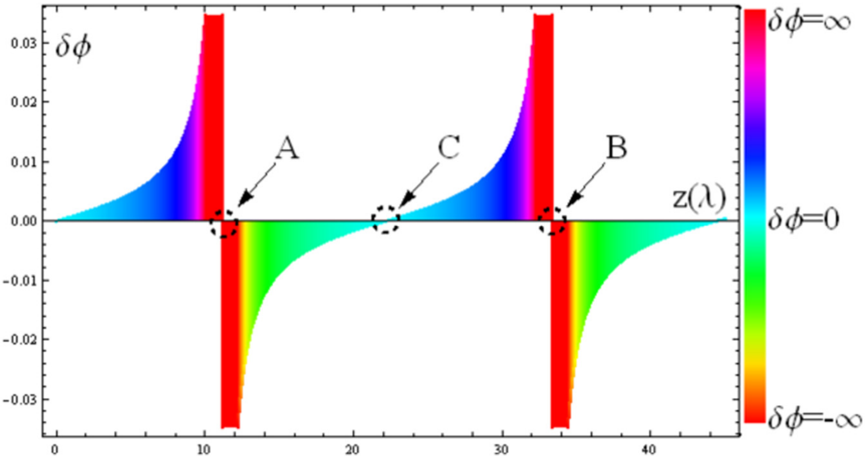

Fig. 1. Evolution of the phase difference δ_ϕ in a propagation period. A and B indicate the phase transition locations, and C indicates the location where the self-acceleration of the cosh-Airy beam is inverted.

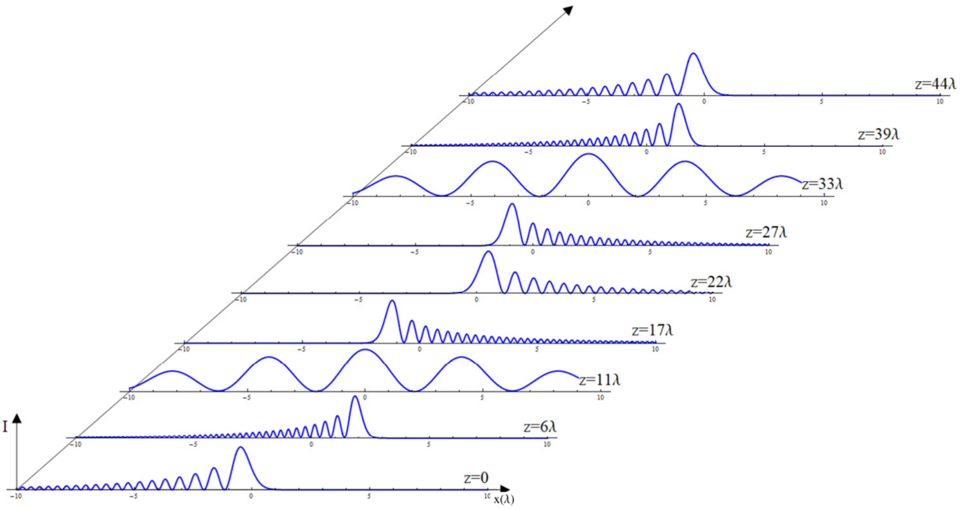

Fig. 2. Evolution of the normalized intensity distribution of the cosh-Airy in a propagation period. The calculation parameters are the same as those used for Fig. 1.

where A, B, D are the elements of the optical metric and are determined by the optical medium. $u(x_1, z = 0)$ is the field distribution of the cosh-Airy beam in the initial plane:

$$u(x_1, z=0) = Ai\left(\frac{x_1}{w_1}\right)\exp\left(\frac{ax_1}{w_1}\right)\cosh\left(\frac{a'x_1}{w_2}\right), \tag{2}$$

where $Ai(.)$ is the Airy function, $\cosh(.)$ is the cosh function, a is the decay factor, a' is the parameter associated cosh function, w_1 is an arbitrary transverse scale of the beam, and w_2 is an arbitrary transverse scale associated cosh function. Because $\cosh(a'x/w_2) = [\exp(a'x/w_2) + \exp(-a'x/w_2)]/2$, the field distribution of the cosh-Airy beam in the initial plane can be written in another form

$$u(x_1, z=0) = \frac{1}{2}\left[Ai\left(\frac{x_1}{w_1}\right)\exp\left(\frac{a_+x_1}{w_1}\right) + Ai\left(\frac{x_1}{w_1}\right)\exp\left(\frac{a_-x_1}{w_1}\right)\right], \tag{3}$$

where $a_\pm = a \pm a'w_1/w_2$. Thus, the cosh Airy-beam can be considered as a superposition of two Airy beams with different decay factors. When the cosh-Airy beam propagates in free space, $A = D = 1$, $B = z$ and $C = 0$, and the propagation properties of cosh-Airy beams are easily obtained [30]. Considering that the cosh-Airy beam propagates in a quadratic-index inhomogeneous medium with the refractive-index distribution $n(x) = n_0(1 - \alpha x^2)$, the matrix ABCD has the following form:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(z\sqrt{2\alpha}) & \sin(z\sqrt{2\alpha})/\sqrt{2\alpha} \\ -\sqrt{2\alpha}\sin(z\sqrt{2\alpha}) & \cos(z\sqrt{2\alpha}) \end{pmatrix}, \tag{4}$$

where α is a coefficient of gradient refractive index. Using the integral definition of the Airy function

$$Ai(x) = \frac{1}{2\pi}\int \exp\left(-\frac{i}{3}u^3\right)\exp(-ixu)du, \tag{5}$$

and the general property of the optical matrix

$$AD - BC = 1, \tag{6}$$

by substituting Eq. (3) into Eq. (1), the analytical expression of the cosh-Airy beam in the quadratic-index inhomogeneous medium is obtained:

$$u(x, z) = \frac{1}{2}[u_{a_+}(x, z) + u_{a_-}(x, z)], \tag{7}$$

where

$$\begin{aligned} u_{a_\pm}(x, z) = {} & \sqrt{b}\exp\left[-ikK\alpha x^2\right] Ai\left[\frac{bx}{w_1} - \left(\frac{K}{2kw_1^2}\right)^2 + \frac{ia_\pm K}{kw_1^2}\right] \\ & \times \exp\left[\frac{a_\pm bx}{w_1} + i\left(a_\pm^2 + \frac{bx}{w_1}\right)\frac{K}{2kw_1^2} - \frac{i}{12}\left(\frac{K}{kw_1^2}\right)^3 \right. \\ & \left. - \frac{a_\pm}{2}\left(\frac{K}{kw_1^2}\right)^2\right]. \end{aligned} \tag{8}$$