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Zonal wavefront reconstruction of Shack–Hartmann and Hartmann patterns with hexagonal cells

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ABSTRACT

In this article we will develop a method to integrate Shack–Hartmann and Hartmann pattern with hexagonal cells, using a polynomial representation (modal integration) over each hexagonal cell. Since each hexagonal has six sampling points, one at each vertex, instead of the typical four sampling points in square cells, it is possible to have a different representation of the wavefront in each cell, each with different aberration terms. The local curvatures and low order aberrations in each cell are calculated more accurately than for square cells. All the analytical functions over each hexagonal cell have a different unknown piston term, that is calculated with a method to be described here. As a result, wavefront retrieval and representation of freeform optical surfaces for some optical systems can be made, due to the calculation of aberrations in each hexagonal cell.

1. Introduction

Wavefront retrieval has become a common step in modern optical testing. When using the Hartmann [1] or Shack–Hartmann [2] test, there are several wavefront retrieval or reconstruction techniques that have been developed, with different advantages and disadvantages all of them. They can be classified in modal and zonal methods. Modal methods include a fitting of the whole wavefront in the pupil to a polynomial, frequently in the form of a Zernike polynomial. If some local deformations with relatively high spatial frequency components are present, the polynomial representation smoothens sharp details at the wavefront, losing most information about these deformations, depending on how many modes are used in the reconstruction. Zonal integration methods do not use a polynomial representation. The most common is the Newtonian or trapezoidal integration [3]. Most of these methods produce very good results if the surface or wavefront is nearly a sphere, but if local small deformations are present, they cannot be obtained and the result does not have a good accuracy. The reason is that if the sampling points separation is large, when integrating between two different sampling points, along different paths, the result is not exactly the same.

Zonal methods using a local polynomial representation in an array of square cells had been proposed by the authors [4]. This representation has several advantages. Since the polynomial representation is not global, but zonal, small local deformations are better represented. Even free form surfaces can be represented with this method. Another advantage is that local curvatures are directly obtained for each sub aperture cell.

In this work we propose a kind of modal method within a zonal representation, since a polynomial or modal representation is independently made inside of each cell that covered the whole pupil, but instead of squares as in our previous publication [4], we propose to use an array of hexagons. This arrangement provides a higher density of spots [5–7] than in an array of square cells, assuming that the lenslets in Shack–Hartmann or the holes in Hartmann test are of the same diameter. Fig. 1 shows two circular pupils with the same diameter, and lenslets or holes also with the same diameter. Furthermore, Fig. 1 shows that if we choose a maximum diameter of the hole or lenslet, the hexagonal array allows to cover a larger area from the pupil than the square array. As in our previous work [4,8] mentioned, the square array is the simplest one, but with it we can obtain the tilts in two orthogonal directions, the spherical power and the astigmatism components. The hexagonal array configuration is complicated to analyze due to the geometry, but it increases the number of aberrations coefficients that can be calculated. For a hexagonal cell of a given cell size, we have two possibilities, either, the located sampling points at the center of the cells or at the vertices. In the first case, the holes or lenslets almost cover the whole pupil, while in the second case there are not sampling points at the center of the cells. However, given a hexagonal cell size, there is a higher sampling

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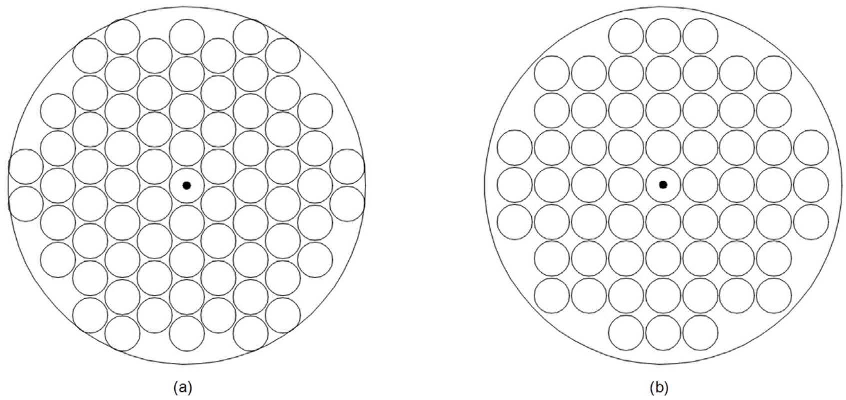

Fig. 1. (a) Hexagonal distribution with a maximum diameter of the holes. (b) Square distribution with the same maximum diameter of the holes that in the hexagonal array.

density if the sampling points are at the vertices and not at the center of the cells. We will use this configuration. If we decided to place another sampling point at the center of the hexagonal cell, our analysis would have to consider a screen with triangular cells instead of hexagonal cells.

In the proposed method, the hexagonal array generated by the Hartmann screen covers the whole pupil and hexagonal cells can be formed. After the hexagonal cells were created, a particular analytical expression for each cell is generated in an exact manner, using the twelve data available (the two slopes at each vertex), as a previous papers [8,9]. By using hexagonal cell, plus the coefficients obtained from a square cell, coma aberrations and triangular astigmatisms can be found.

It is well-known that the Hartmann screen test method measure the local wavefront slopes, transverse aberrations as they are commonly known, thus, an integration method is necessary to make the wavefront reconstruction by converting gradients to phase. For this reason, a huge number of methods to wavefront retrieval have been created and classified in two types, modal and zonal. One of the most commonly used zonal methods retrieves the wavefront by Newtonian or trapezoidal integration. However, a disadvantage is that if the optical surface or wavefront to be measured has strong aberrations with wavefront high spatial frequency components, the local slopes can change abruptly and, the retrieved wavefront cannot contain the high spatial frequency components.

Research on wavefront retrieval techniques has been continuously developing due to the improving accuracy for methods for wavefront sensing. A reason is that an important application of wavefront sensing is the use in the study of human eye. The fact that low-order aberrations can be measured and corrected in clinical practice, but since some high-order aberrations, such as coma and spherical aberrations, are difficult to correct and measure, it is important to continue the research in this area [10]. Visual personalized correction is the objective that new research in wavefront sensing is aimed to develop new methods and techniques. Wavefront aberrations can be used to describe the optical properties of the human eye, and the Shack–Hartmann sensor is a common technique used by aberrometers to obtain the ocular aberrations [11].

The addition of freeform optical surfaces to an optical system is another reason to maintain the research in wavefront analysis techniques. A freeform surface can be interpreted as a non-rotationally symmetric surface [12] and which can reduce the number of elements, decrease the aberrations and the size of the system [13]. The problems with the freeform optical surfaces are their fabrication and testing, and sometimes the cost of manufacturing [12–14]. Many efforts have been realized to make algorithms and tests that can solve the issues of freeform optical surfaces test [12,14].

As can be seen, several methods and techniques have been developed in recent years, aimed to increase the wavefront retrieval based on Shack–Hartmann test [11,15]. Also, spline techniques [16,17] have been used to enhance wavefront reconstruction. Our work presented in this paper can be similar to the spline methods, but the main difference is the basis used to represent the individual polynomial of each cell. Another difference is the use of a hexagonal array of sampling points that we use to generate the wavefront retrieval. In our case, in this paper we change the array geometry of the lenslets proposed in our previous work. The developed method of wavefront retrieval using hexagonal cells is described in the following sections. Also, it is necessary to add a previous analysis to design the distribution of the lenslets in the hexagonal array and the way how can be designed the Hartmann plate.

2. Coordinates for the center and vertices of the hexagonal cells

Given an array of hexagons, the center of the holes or lenslets, which produce the sampling points by focusing, can be placed at the center of each hexagon or at each vertex. In our case we place the holes or array in a configuration which allows to generate a hexagonal cell at the center of the pupil.

When performing the integration, the location of each hexagonal cell within the whole cell array inside the circular pupil has to be described by a pair of numbers. However, to use Cartesian coordinates or polar coordinates becomes complicated. A simpler system is proposed, as in Fig. 2, where we need two numbers to define the location of any hexagonal cell. A hexagonal cell is at the center of the pupil, with hexagonal rings of cells around this central cell with numbers $n = 1$ to N. Each ring has n hexagonal cells on each side, so that the total number of cells in the ring is $6n$. Finally, all cells at any hexagonal ring are numbered with $m = 1$ at the first cell of the first side, with a maximum equal to $6n$. These definitions can be more clearly seen in Fig. 2.

The size of each hexagonal cell is determined by its side length s or its apothem a, which are related by:

$$a = \frac{\sqrt{3}}{2} s \tag{1}$$

We define an auxiliary parameter k representing the side number for the hexagonal ring, where $k = 1$ for the first side at the right and in the upper part of the pupil. It can be shown that given a pair of values n and m, the value m begins at the x axis, increasing its value in an counter clockwise direction. The value of k is given by:

$$k = int\left[\frac{m-1}{n-1}\right] + 1 \tag{2}$$

where *int* is the non-rounded integer value.

It is now convenient to express the location of a hexagonal cell in a ring n, with a number m'_k instead of the number m. The difference is that the beginning for the number m is at the first hexagonal cell of the whole hexagonal ring, while the origin for number m'_k is at the first cell