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Aggregate capacity optimization algorithm for large pool size multi-mode orbital angular momentum free space optical beam communication

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ABSTRACT

This paper proposes a simple iterative algorithm to determine the optimum (in terms of the system capacity) set of multiple orbital angular momentum (OAM) modes over Free space optical (FSO) communication beam for a given receiver radius using generalized channel efficiency matrix. The optimum set is the same as the obtained set from exhaustive search that was previously proposed in the literature but has a much lower complexity. The proposed algorithm is especially efficient for large number of modes. The complexity reduction is in the order of $\left(2^{\mathbf{N}}\right)/\mathbf{N}^2$ when compared to the exhaustive search, where N is the pool size of available modes.

1. Introduction

Several studies have derived the capacity of orbital angular momentum modes (OAM) beams as cross-talk probability distribution using the received power model of OAM beams under different models of turbulence either Kolmogorov as in [1] or non-Kolmogorov as in [2–4]. Others have determined the equivalent channel matrix which describes the mode propagation and derives the channel efficiency matrix representing the power correlation between the different modes under turbulence [5,6]. This enabled the equivalence between the system in that case and the multiple-input multiple-output (MIMO) system to be established; therefore, equalization and compensation techniques from the MIMO literature are performed to enhance the capacity [7]. However, the resulting capacity form encompasses multiple parameters that make it intractable for further system analysis.

Following the recent literature focus concerned with the configurable generation of modes [8–10], it has been feasible to flexibly generate as much modes as possible aiming at increasing the achievable rates. However, unregulated increase in modes may cause deteriorated performance due to the increasing cross-talk among modes which leads to decrease in the achieved rates. For the above reason, it became necessary to find a mechanism to guarantee we are using the suitable modes for the highest communication gain that can justify the added cost of generating and combining several modes. To the authors' best knowledge, there have not been any methodology to find the best operating modes in OAM system except by trial and error through extensive simulations to run exhaustive search algorithm, e.g., [6]. Analytically, we shall show that it is an unsolvable decision problem.

Practically, the exhaustive search for the best transmission modes may not be feasible in the presence of large number of available modes in a system that is described by several parameters such as the turbulence, the transmitter–receiver misalignment model, as well as the number of modes and beams. This motivates the present work which proposes a simple novel iterative algorithm to obtain the optimum set of OAM modes for a given receiver's radius in a beam propagating in weak to medium turbulence.

To adequately formulate the modes-optimization algorithm as discussed in the sequel, a complete channel model should be used. Therefore, we have included the cross talk effect resulting from poor receiver alignment that was tackled in [11–14] and put it in a new closed form. For a given total amount of fixed power, leakage mode power, receiver area and misalignment drift, we obtain the optimal set of modes that maximizes the achievable rate of OAM communication system using a simple aggregation algorithm. Compared to the exhaustive search, the proposed algorithm has a much lower complexity and also achieves the same performance as the exhaustive search algorithm. Monte Carlo simulations are provided to show that the optimal sets are properly found.

The paper is organized as follows. Section 2 presents the system model. Section 3 presents the novel simple optimization algorithm for selecting the best OAM modes. Finally, Section 4 discusses the algorithm's optimality and the conclusion follows in Section 5.

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2. System model

Consider a Laguerre–Gaussian (LG) beam carrying single mode that has the parameters defined in [2,15] with zero-order Laguerre function carrying OAM mode 'm' affected by turbulence which has the model defined in [4].

Assume that the beam is carrying the data signal denoted by $S_m(t)$ over this single mode 'm'. The noise-free received waveform (denoted by U_m) for single mode is written as:

$$U_m = S_m(t)\sqrt{\frac{2}{\pi\, w(z)^2 (|m|)!}}\left(\frac{\sqrt{2}r}{w(z)}\right)^m . e^{-\frac{(r^2)}{w(z)^2}} e^{im\theta} e^{i(kz-\omega t)} . e^{-\frac{ik(r^2)}{2R_c(z)}} e^{-i\varphi(z)} e^{\psi} \quad (1)$$

Consider multiple 'M' beams where each beam is a single-mode beam. Each beam is modulated with a single data stream (the data streams are independent from one beam to another). The beams are then collimated and coaxially multiplexed, therefore the resultant received beam carry multiple modes. The beam can be written as:

$$y = \sum_{i=m(1)}^{m(M)} \sqrt{\frac{E_s}{MP_i}} U_i S_i = h^T S(t) \quad (2)$$

where we put:$h^T = [h_1, h_2, \ldots h_M] = \sqrt{\frac{E_s}{MP_i}}[U_1, U_2, \ldots U_M]$ and $S(t) = [S_1, S_2, \ldots S_M]$.

We dropped the time variable 't' for simplicity. $R_c(z) = z + \frac{z_R{}^2}{z}$ is the radius of curvature, beam width at transmitter w_0 and at receiver: $w(z) = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2}$, $z_R = \frac{w_0{}^2\pi}{\lambda}$ is the Rayleigh range, $\varphi(z) = \arctan\left(\frac{z}{z_R}\right)$ is the Gouy phase, and λ is the beam wavelength. E_{si} is the symbol's average energy for data carried on mode 'i' and P_i is the total power of mode 'i' which is equal for all of the used M modes. Following the above model, each mode's power equals to unity; $P_i = 1$. M is the actual number of the utilized modes: $\{m(1), \ldots, m(M)\} = \{m_1, \ldots, m_M\}$.

The exponential term e^{ψ} represents the first order Rytov approximation for the turbulence induced by the channel. It is generally a complex random variable that can be approximated by the lognormal distribution in weak to moderate turbulence [15,16]. We have used non-Kolmogorov generalized Von-Karman model for the spatial frequency spectrum $\phi_n(K)$ of turbulence described in [6] as:

$$\phi_n(K) = 0.033k^2 LC_n^2\left(1 + \frac{1.802K}{K_l} - 0.254\left(\frac{K}{K_l}\right)^{\frac{7}{6}}\right) . (K^2 + K_m{}^2)^{-11/6}\left[\exp(-\left(\frac{K}{K_l}\right)^2)\right] \quad (3)$$

where L is the propagation distance, K_l inner scale wave number ($=3.3l_0$, l_0 is the inner scale turbulence), K_m outer scale wave number ($=5.92l_0$), C_n^2 is the medium refractive index structure function described in [15] its value ranges from 10^{-12} for strong turbulent medium to 10^{-16} for weak turbulence. The turbulence experienced through the beam propagation is modeled at the receiver using a turbulence phase screen. The phase screen ψ is a matrix that adds the equivalent turbulence phase and amplitude to each point in the incidence plane perpendicular to the beam propagation at the receiver and is a function of the space coordinates (Cartesian [x,y,z] or polar: [r,θ,z]) and is obtained as in [17]:

$$S(x,y) = \mathrm{Re}\left\{\mathrm{IFFT}\left\{[C + iD]\left[\frac{2\pi}{N\Delta x}\right] k\sqrt{2\pi\Delta z\phi_n(k_x, k_y)}\right\}\right\} \quad (4)$$

where $\Delta x = \Delta y$ are screen resolution, K is the spatial frequency used, k_x, k_y are the spatial frequencies in the x, y directions, N is the number of elements in x or y directions, C and D are $N \times N$ matrices of normal distribution values of zero mean and unit variance. The above expression for the phase screen can be easily expressed in polar coordinates through a transformation matrix D to obtain the corresponding $\psi(r,\theta) = DS(x,y)$ that is used in Eq. (1).

The term $e^{im\theta}$ represents the orbital momentum for mode 'm'. The waveforms for different 'm' are mutually orthogonal [6,18].

At the receiver, the beam is power divided into M branches, each branch is then matched with one of the beam modes—matching with $e^{im\theta}$. Ideally, in turbulence-free environment, only the target mode is received at the output of its receiver branch. However; in the presence of the turbulence, all the received modes' branches output non-zero power for every sent mode due to leakage from each sent mode to all others. The target mode dominance is reduced as the turbulence strength increases. The channel estimation is performed by sending pilot single mode beam and correlating with $e^{-in\theta}$ at the receiver [18] (for every mode n) followed by an integration process over the receiver area [6]. Thus for receiver radius 'R', the output of the nth correlator (matching with mode 'n') for the transmit mode 'm' is:

$$\begin{aligned}\overline{y_{nm}} &= \iint_{-x1,-y1}^{x1,y1} (y + N_n)\, e^{-in\theta} dxdy \\ &= \iint_{0,0}^{R,2\pi} (y + N_n)\, e^{-in\theta} r dr d\theta \\ &= \sqrt{|P_{nm}|} S_m + \bar{N}_n \end{aligned} \quad (5)$$

where $|P_{nm}|$ is the leaked power from transmit mode m to mode n. The parameters $[P_{nm}]$ form a matrix that is known as the channel efficiency coefficients matrix [6]. N_n is the thermal noise added at receiver branch 'n' with noise power σ_{Nn}^2. Using the above receiver model, we can obtain the ratio of the signal power to the interference and noise power for mode 'i' (denoted by SINRi) and hence, the aggregate capacity of the multi-mode beam carrying the modes: $m(1), m(2) \ldots m(M)$ is [19]:

$$\begin{aligned} C &= \sum_i \log_2(1 + \mathrm{SINR}_i) \\ &= \sum_{i=1}^{r} \log_2\left\{1 + \frac{E_{si}|h_i|^2}{\sum_{n\neq i} E_{sn}|h_n|^2 + \sigma_n^2}\right\} \\ &= \sum_{m=m(1)}^{m(M)} \log_2\left(1 + \frac{P^M{}_{mm}}{\left\{\sum_{\substack{n=m(1)\\ n\neq m}}^{m(M)} P^M{}_{mn}\right\} + \sigma_{Nm}^2}\right) \end{aligned} \quad (6)$$

the superscript 'M' indicates we are in an M-modes operation, for turbulence free media, we have:

$$\sum_{\substack{n=m(1)\\ n\neq m}}^{m(M)} P^M{}_{mn} = 0$$

and the capacity is increased by increasing the number of modes; however, in fixed total power transmission scheme, adding modes decreases each mode's power to keep a fixed total power. Overall, the capacity is practically limited by: (1) the total fixed beam power, (2) the turbulence strength, and (3) the receiver size. In the case of non-zero turbulence, the different modes cause interference to each other and therefore limit the achieved data rate. Using Eq. (4), we can guarantee a monotonic increase (with the number of modes) of the capacity by enforcing the sum interfering modes power to be less than the power of the intended mode (for every mode) if:

$$\frac{\mathrm{SNR_d}}{\sigma_{Nm}^2}\left\{1 - \sum_{\substack{n=m(1)\\ n\neq m}}^{m(M)} \frac{P^M{}_{mn}}{P^M{}_{mm}}\right\} > M \quad (7)$$

or equivalently

$$\sum_{\substack{n=m(1)\\ n\neq m}}^{m(M)} P^M{}_{mn} < P^M{}_{mm} \forall m. \quad (8)$$