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Crosstalk probability of the bandwidth-limited orbital angular momentum mode of Bessel Gaussian beams in marine-atmosphere turbulence

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ABSTRACT

The diffraction of the beam edges affects the transmission of the orbital angular momentum (OAM) mode carried by vortex beam. Based on the bandwidth-limited OAM orthogonal vortex basis set, we study the crosstalk probability of the OAM mode carried by the Bessel Gaussian beam in marine-atmosphere turbulence. We find that the crosstalk probability of the bandwidth-limited OAM mode is larger than that of the spiral plane mode. The crosstalk probability of the bandwidth-limited OAM mode decreases with the increment of the inner scale of turbulence, but this crosstalk probability increases with the increment of the outer scale of turbulence. A Bessel Gaussian beam with optimum initial radius of beam, and long beam wavelength in the transmission window is preferable for application. The energy of the OAM mode primarily spreads to adjacent state. Our research confirms that the beamwidth factor of OAM mode should be added to the study of free-space optical communication to make the calculation results closer to the realistic situations.

1. Introduction

In recent years, the propagation of orbital angular momentum (OAM) mode through turbulence media has become a great interesting issue because of their potential applications in the free-space optical (FSO) communication as well as their interesting properties [1–6]. Numerous studies have attempted to explain the propagation characteristics of OAM mode of vortex beam in atmospheric turbulence. Based on the extended Huygens–Fresnel principle, K. Zhu et al. explored the properties of Bessel Gaussian beams (BGBs) propagation in the atmosphere turbulent [7]. For the partially coherent BGBs propagation, B. Chen et al. found that the beams with higher source coherence can be more influenced by atmospheric turbulence than those of lower coherence [8]. Y. Zhu et al. explored the behavior of OAM modes of the partially coherent modified BGBs (PCMBG) propagating in the anisotropic and non-Kolmogorov turbulence of marine-atmosphere, and they found that the OAM mode of PCMBG beams with long wavelength, low quantum number, and a high spectral degree of coherence of the source have stronger anti-turbulence interference ability [9]. J. Gao et al. developed the models of average probability densities and the normalized powers of signal/crosstalk OAM modes for the fractional BGBs in the turbulence atmosphere of strong irradiance fluctuations. They demonstrated that the increase of turbulence fluctuations can make the crosstalk stronger and more concentrated. Lower irradiance fluctuation can give rise to higher normalized powers of the signal OAM modes [10]. We know that, in the transmission of the light beam, the beam edges which are similar to an aperture will produce the diffraction of the light beam. The diffraction of the beam edges causes the beam spreading and the spreading will affect the transmission performance of optical signals. Therefore, it is deserved to consider the influence of the diffraction of the beam edges when we study the OAM mode of vortex beam propagating in atmospheric turbulence. However, to the best of our knowledge, there has a little discussion about the effect of the beam edges diffraction on the transmission of the OAM mode in marine-atmosphere turbulence.

In this paper, we put forward a model of the crosstalk probability of the bandwidth-limited OAM mode carried by BGBs. By studying the crosstalk probability, we research the effect of the beam edges diffraction on the vortex beam transmitting in marine-atmosphere turbulence. This paper is organized as follows. In Section 2, we derive the bandwidth-limited OAM orthonormal vortex basis set. In this paper, the term "the bandwidth-limited OAM orthonormal vortex basis set" refers to the OAM orthonormal vortex basis set including the factor of initial radius of the beam. In Section 3, we derive the expression of the crosstalk probability of the bandwidth-limited OAM mode carried by BGBs. The impacts of the beam edges diffraction and the channel

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parameters of optical communication links on crosstalk probability are given in Section 4, and conclusions are given in Section 5.

2. The bandwidth-limited orthonormal vortex basis set

In optical vortex beam research, the limited transverse spatial coherence of practical light sources means that radius vortex modes defined using a circular aperture (or pupil) function are more appropriate in the realistic situations. The simplest bandwidth-limited vortex beam can be generated by the Fraunhofer diffraction of a plane wave by a spiral phase plate with the transmission function

$$S_m(r,\varphi,z)=\exp\left(\mathrm{i}m\varphi+\mathrm{i}k_z z\right), \tag{1}$$

through an transmitting aperture of the radius ω_0, it is also be called initial radius of the beam. Here (r,φ,z) are cylindrical coordinates, r is the radial distance from the propagation axis, φ is the azimuth angle and z is the distance along the propagation axis. i is the complex symbol, m is OAM quantum number and k_z is the component of the wave number vector along in z axis. To represent the wave form of the arbitrary beam in similar bandwidth-limited situations, an orthonormal vortex basis set characterized by OAM was recently reported for vortex beams [11], including both the azimuthal and radial quantum numbers m and p, respectively. That is

$$S_{mp}(r,\varphi,z)=N_p^m\exp\left(\mathrm{i}m\varphi+\mathrm{i}k_z z\right)\mathrm{J}_m\left(k_\perp^{pm}r\right). \tag{2}$$

In Eqs. (2), $N_p^m=1/\left[\sqrt{2}\pi\mathrm{J}_{m+1}\left(\lambda_{pm}\right)\right]$ is the normalization factor [11], $k_\perp^{pm}=\lambda_{pm}/\omega_0$ is the magnitude of the transverse wave vector, λ_{pm} is the $(p+1)$th zero of the mth-order Bessel function and J_m is the mth-order first kind Bessel function [11]. Substituting $N_p^m=1/\left[\sqrt{2}\pi\mathrm{J}_{m+1}\left(\lambda_{pm}\right)\right]$ into Eq. (2), we have the bandwidth-limited OAM orthonormal vortex basis set

$$S_{mp}(r,\varphi,z)=\frac{1}{\sqrt{2}\pi\mathrm{J}_{m+1}\left(\lambda_{pm}\right)}\exp\left(\mathrm{i}m\varphi+\mathrm{i}k_z z\right)\mathrm{J}_m\left(k_\perp^{pm}r\right). \tag{3}$$

3. The crosstalk probability of the bandwidth-limited OAM mode carried by BGBs

It is know that under the conventional Rytov approximation, the complex amplitude of BGBs is expressed by [12]

$$u_m(r,\varphi,z)=u_{m_0}(r,\varphi,z)\exp\left[\psi(r,\varphi,z)\right], \tag{4}$$

where ψ is the complex phase perturbations caused by the turbulence, m_0 is an integer corresponding to the initial OAM quantum number and $u_{m_0}(r,\varphi,z)$ is BGBs with OAM quantum number m_0 at the z plane in the absence of turbulence. The $u_{m_0}(r,\varphi,z)$ in Eqs. (4) can be expressed as [13]

$$\begin{aligned}u_{m_0}(r,\varphi,z)=\;&A_0\frac{\omega_0}{\omega(z)}\mathrm{J}_{m_0}\left(\frac{k_r r}{1+\mathrm{i}z/z_0}\right)\\&\times\exp\left[\mathrm{i}\left(k-\frac{k_r^2}{2k}z-\zeta(z)+\frac{-1}{\omega^2(z)}\right)\right]\\&\times\exp\left[\left(\frac{\mathrm{i}k}{2R(z)}\right)\left(r^2+k_r^2\frac{z_0}{k^2}\right)+\mathrm{i}m_0\varphi\right],\end{aligned} \tag{5}$$

where A_0 is a constant characterizing the beam power, $k=\left(k_z^2+k_r^2\right)^{1/2}$ is the wave number related to the wavelength λ, $\omega(z)=\omega_0[1+(z/z_0)^2]^{1/2}$ is the beam radius, $z_0=k{\omega_0}^2/\lambda$ is the Rayleigh range and $\zeta(z)=\tan^{-1}\left(z/z_0\right)$ is the Gouy phase. The angular half-aperture of the cone $\theta_C\left(=\omega_0/z\right)$ is related to the radial frequency, k_r, as $k_r=k\sin\left(\theta_C\right)$. Now, the complex amplitude $u_m(r,\varphi,z)$ is written by

$$u_m(r,\varphi,z)=\sum_m\beta_m(r,z)N_p^m\exp\left(\mathrm{i}m\varphi+\mathrm{i}k_z z\right)\mathrm{J}_m\left(k_\perp^{pm}r\right). \tag{6}$$

The expansion coefficient is given by the integral [14]

$$\beta_m(r,z)=\frac{N_p^m}{2\pi}\exp\left(-\mathrm{i}k_z z\right)\int_0^{2\pi}u_m(r,\varphi,z)\exp(-\mathrm{i}m\varphi)\mathrm{J}_m^*\left(k_\perp^{pm}r\right)\mathrm{d}\varphi. \tag{7}$$

Instead of the random variable $\beta_m(r,z)$, we are usually interested in the ensemble average over the turbulence statistics [15], i.e.

$$\begin{aligned}\langle|\beta_m(r,z)|^2\rangle=\;&\left(\frac{N_p^m}{2\pi}\right)^2\left|\mathrm{J}_m\left(k_\perp^{pm}r\right)\right|^2\int_0^{2\pi}\int_0^{2\pi}u_{m_0}(r,\varphi,z)\\&\times u_{m_0}^*\left(r,\varphi',z\right)\exp\left[-\mathrm{i}m\left(\varphi-\varphi'\right)\right]\\&\times\exp\left\{-\rho_0^{-2}\left[2r^2-2r^2\cos\left(\varphi-\varphi'\right)\right]\right\}\mathrm{d}\varphi\mathrm{d}\varphi',\end{aligned} \tag{8}$$

where ρ_0 is the spatial coherence radius of a spherical wave propagating in turbulence and given by [12]

$$\rho_0=\left[\frac{\pi^2k^2z}{3}\int_0^\infty\kappa^3\phi_n(\kappa)\,\mathrm{d}\kappa\right]^{-1/2}. \tag{9}$$

In Eq. (9), $\phi_n(\kappa)$ is the turbulent spectrum of the marine atmosphere and is given by [16]

$$\begin{aligned}\phi_n(\kappa)=\;&\frac{0.033C_n^2}{\left(\kappa^2+\kappa_0^2\right)^{11/6}}\exp\left(-\frac{\kappa^2}{\kappa_H^2}\right)\\&\times\left[1-0.061\frac{\kappa}{\kappa_H}+2.836\left(\frac{\kappa}{\kappa_H}\right)^{7/6}\right],\end{aligned} \tag{10}$$

where κ is the refractive index fluctuation spatial wave number, C_n^2 is the refractive index structure constant at the sea surface with units $\mathrm{m}^{-2/3}$, $\kappa_0=2\pi/L_0$, L_0 is the outer scale of turbulence , l_0 is the inner scale of turbulence and $\kappa_H=3.41/l_0$. Substituting Eq. (10) into Eq. (9) and making use of the following integral formula [17]

$$\begin{aligned}\int_0^\infty\kappa^{2\mu}\frac{\exp\left(-\kappa^2/\kappa_H^2\right)}{\left(\kappa^2+\kappa_0^2\right)^{11/6}}\mathrm{d}\kappa=\;&\frac{1}{2}\kappa_0^{2\mu-8/3}\Gamma\left(\mu+\frac{1}{2}\right)\\&\times\mathrm{U}\left(\mu+\frac{1}{2};\mu-\frac{1}{3};\frac{\kappa_0^2}{\kappa_H^2}\right),\mu>-\frac{1}{2},\end{aligned} \tag{11}$$

we have

$$\begin{aligned}\rho_0^{-2}=\;&C_n^2\pi^2k^2z\left[0.0055\kappa_0^{1/3}\Gamma(2)\,\mathrm{U}\left(2;\frac{7}{6};\frac{\kappa_0^2}{\kappa_H^2}\right)\right.\\&-0.0003355\frac{\kappa_0^{4/3}}{\kappa_H}\Gamma\left(\frac{5}{2}\right)\mathrm{U}\left(\frac{5}{2};\frac{5}{3};\frac{\kappa_0^2}{\kappa_H^2}\right)\\&\left.+0.015598\frac{\kappa_0^{3/2}}{\kappa_H^{7/6}}\Gamma\left(\frac{31}{12}\right)\mathrm{U}\left(\frac{31}{12};\frac{21}{12};\frac{\kappa_0^2}{\kappa_H^2}\right)\right],\end{aligned} \tag{12}$$

where $\mathrm{U}(a;b;z)$ is the confluent hypergeometric function of the second kind and $\Gamma(a)$ is the gamma function. Following Eqs. (8)–(12) and using the following integral expression [17],

$$\int_0^{2\pi}\exp\left[-\mathrm{i}n\varphi_1+x\cos\left(\varphi_1-\varphi_2\right)\right]\mathrm{d}\varphi_1=2\pi\exp\left(-\mathrm{i}n\varphi_2\right)\mathrm{I}_n(x), \tag{13}$$

we can obtain

$$\begin{aligned}\langle|\beta_m(r,z)|^2\rangle=\;&\left(A_0N_p^m\right)^2\frac{\omega_0^2}{\omega^2(z)}\left|\mathrm{J}_m\left(k_\perp^{pm}r\right)\right|^2\\&\times\exp\left[-2\zeta(z)+\frac{-2}{\omega^2(z)}-\frac{2r^2}{\rho_0^2}\right]\\&\times\left|\mathrm{J}_{m_0}\left(\frac{k_r r}{1+\mathrm{i}z/z_r}\right)\right|^2\mathrm{I}_{m-m_0}\left(\frac{2r^2}{\rho_0^2}\right),\end{aligned} \tag{14}$$

where $\mathrm{I}_n(\cdot)$ is the Bessel function of second kind with n order. With the help of Eqs. (5), (12) and (14), we have the average intensity of the