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Simultaneously estimating two phases in three-mode nonlinear interferometer

Ying-Wang [a], Li-Yun Hu [b], Zhi-Ming Zhang [c], Chao-Ping Wei [a,*]

[a] Jiangxi Province Key Laboratory of Water Information Cooperative Sensing and Intelligent Processing, Nanchang Institute of Technology, School of Information Engineering, Nanchang Institute of Technology, Nanchang 330022, China
[b] Center for Quantum Science and Technology, Jiangxi Normal University, Nanchang 330022, China
[c] Guangdong Provincial Key Laboratory of Nanophotonic Functional Materials and Devices (SIPSE), South China Normal University, Guangzhou 510006, China

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ABSTRACT

We propose a multi-mode nonlinear interferometers structure by cascading several multi-mode beam splitters (MBS) and nonlinear phase shifters (NPS), in which multiple phases can be estimated simultaneously. Specially, we mainly discuss two phases estimation in a three-mode nonlinear interferometer with single photons as inputs. The phase sensitivity of the three-mode nonlinear interferometer can be discussed by the quantum Fisher information matrix (QFIM) method after the states undergo the nonlinear phase shifters, and can also be studied by classical Fisher information matrix (CFIM) method in the interferometer's output ports. It is shown that the precision given by CFIM can achieve the bound of QFIM. Finally, we make a comparison with an optimal probe state in terms of the total variance and discuss the relationship of the total variance among three different phase estimation schemes.

1. Introduction

Quantum metrology is one of the most fascinating frontiers in measurement science, and the phase estimation is one of the most important issues in quantum metrology and has been widely studied [1]. Optical interferometers are the main devices for metrology and weak signal detection. In the early days, the physicists mainly pay attention to single phase estimation, which can be executed in conventional two-mode interferometer, such as linear Mach–Zehnder interferometer or nonlinear SU(1,1) interferometer. It is shown that the phase sensitivity of interferometers can be improved by inputting some entangled [2] and squeezed states [3,4] or with the help of the nonlinearity [5,6]. Experimentally, the role of entanglement has been investigated in an Mach–Zehnder interferometer [7], and the signal-to-noise ratio of the SU(1,1) interferometer has been improved greatly with squeezed light as inputs [8].

However, in some realistic environment, it is very necessary to estimate multiple phases simultaneously, such as optical or magnetic field sensing, quantum imaging and gravitational wave astronomy [9,10]. Recently, multi-parameter estimation theory get a rapid development, especially in the multiple phases estimation. The theory model of the multiple phases estimation can be found in Refs. [11,12]. Besides, it is shown that, given a fixed total photon number, the precision of estimating multiple phases outperforms individual quantum estimation schemes. The multiple phase estimation problem has also been investigated for a natural parametrization of arbitrary pure states under white noise [13]. Experimentally, the multiple phases estimation has usually been studied in some multi-arm interferometers [14], and the precision of multiple phases estimation can still beat the shot-noise limit even in the presence of loss [15]. Recently, many multi-mode interferometers have been studied by combining the phase shifters and multi-mode beam splitters [14,16], and in which several phases can be estimated simultaneously. In addition, the nonlinear phase shifter has been involved in Refs. [17–21] and the phase sensitivity can achieve a super-Heisenberg limit with the help of the nonlinear phase shifter [18,19]. Based on the above descriptions, in our paper, we shall combine the multi-mode interferometer and the nonlinear phase shifter to improve the phase sensitivity of interferometers at the same time.

In this paper, we propose an architecture of multi-mode nonlinear interferometers by combining several multi-mode beam splitters and nonlinear phase shifters. In this interferometer, several phases can be estimated simultaneously. To be specific, we mainly study the phase sensitivity of a three-mode nonlinear interferometer with single photons as inputs. For analysis of output states in the three-mode nonlinear interferometer, QFIM method is used to estimate two phases simultaneously

* Corresponding author.
E-mail address: weichaoping0@163.com (C.-P. Wei).

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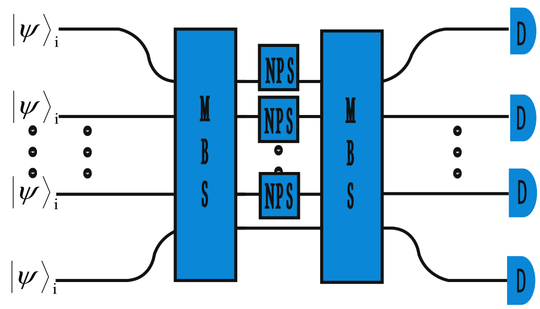

Fig. 1. Sketch of a multi-mode nonlinear interferometer with M input states $|\psi\rangle_i$ as inputs. The M input states inject into the first multi-mode beam splitters (MBS), respectively. Applying a nonlinear phase shifter (NPS) $U(\phi,k)$ to M-1 modes of the path, the Mth mode is our reference mode, and then these light beams combining in the second multi-mode beam splitters. Finally, we can execute a photon-number detection in each output port.

after the states undergo the nonlinear phase shifters. It is proved that it is impossible to achieve the single phase case unless the off-diagonal elements of QFIM are equal to zero. We also discuss the phase sensitivity of the three-mode nonlinear interferometer by the CFIM in terms of the output measurement probabilities. It is also shown that the phase sensitivity strongly depends on these two phases ϕ_1 and ϕ_2, and the precision given by CFIM can reach the precision bound of QFIM in some special areas. Finally, we make a comparison with an optimal probe state with the same average photon number, and discuss the relationship of the total variance of three different phase estimation schemes.

The organization of this paper is as follows. In Section 2, an architecture of multi-mode nonlinear interferometer is introduced detailedly. In Section 3, we obtain a three-mode nonlinear interferometer by simplifying the multi-mode nonlinear interferometer. In this interferometer, simultaneous estimate of two phases is studied by quantum and classical Fisher information matrix method. A detailed discussion is introduced in Section 4, where we give an optimal input state and make a comparison with our scheme in terms of the total variance. We also make a detailed comparison among the total variance ($\left|\Delta\phi_{sep}\right|^2$) of estimating two phases using two same separate models, the total variance ($\left|\Delta\phi_{ind}\right|^2$) of estimating two phases in one model independently, and the total variance ($\left|\Delta\phi_{sim}\right|^2$ of estimating the phase ϕ_1 and ϕ_2 in one model simultaneously. Section 5 presents the conclusions.

2. Multi-mode nonlinear interferometer

Multi-mode optical interferometers are the key devices for multiple phase estimation, and it can be realized by cascading several multi-mode beam splitters (MBS) and phase shifters [22]. The universal theoretical framework of multi-mode interferometer can be found in Ref. [23,24], and the phase sensitivity of this interferometer is capable of significantly beating the shot noise limit fed with only uncorrelated, single photons inputs. In Ref. [25], the authors directly exploit number-path entanglement generated in quantum Fourier transform interferometers and demonstrate optical phase supersensitivities deterministically. In addition, experimentally, these proposed devices can be implemented with only passive linear optics, single photon sources and on–off or photon detector.

In this section, we propose a multi-mode nonlinear interferometer as shown in Fig. 1. In the multi-mode nonlinear interferometer, the conventional linear phase shifter $U(\phi,1) = e^{i\phi a^\dagger a}$ is replaced by a nonlinear phase shifter $U(\phi,k) = e^{i\phi(a^\dagger a)^k}$ $(k \geq 1)$. Applying a nonlinear phase shifter (NPS) $U(\phi,k)$ to M-1 modes of the path, the M-th mode is our reference mode, this constitutes our multi-mode nonlinear interferometer. In this interferometers, the precision of estimating single or multiple phase can be enhanced greatly with the help of number-path entanglement inside the interferometer and nonlinear phase shifters. In the following, we mainly study the phase sensitivity of a three-mode nonlinear interferometer by simplifying multi-mode nonlinear interferometer.

3. Phase sensitivity of three-mode nonlinear interferometer with single photons inputs

Theoretically, we can estimate arbitrary multiple phases by using multi-mode nonlinear interferometer, but the complexity of interferometers will be increase with the number of modes of the interferometers and the complexity of input states. Therefore, in next section, we only discuss three-mode nonlinear interferometer and consider single photons as inputs. By simplifying multi-mode nonlinear interferometer, we get the three-mode nonlinear interferometer which can be realized by cascading two three-port beam splitters (tritters) and two nonlinear phase shifters. We study the phase sensitivity of the three-mode nonlinear interferometers using the quantum and classical Fisher information matrix method with single photons inputs and only estimate two phases simultaneously.

For a single phase estimation, the precision of phase estimation can be analyzed by quantum Fisher information F_Q (QFI) and quantum Cramer–Rao bound [26], **i.e.** $\Delta\phi \geq \frac{1}{\sqrt{\mu F_Q}}$, where μ is the number of independent measurements. The QFI describes the information associated with phase estimation limited only by the initial quantum states. In the actual physical models, like in some optical interferometers, we need obtain the phase information by proper detection, and the phase sensitivity of these interferometers can be discussed by the error propagation equation or classical Fisher information F_C (CFI) and Cramer–Rao bound ($\Delta\phi \geq 1/\sqrt{\mu F_C}$). The CFI is relevant to the initial quantum states and measurement probabilities, and it is bounded by the QFI ($F_C \leq F_Q$). The goal of phase estimation is to enhance the QFI and CFI as soon as possibly. For multiple phases estimation, we need extend the QFI and CFI to QFIM and CFIM, respectively.

3.1. QFIM method

For multiple phase estimation, the performance of estimating a series of phases $[\phi_1,\phi_2,\cdots\phi_i]$, the quantum Cramer–Rao bound is a lower bound to the covariance matrix $Cov(\phi)$ in terms of the quantum Fisher information matrix (QFIM) F_Q [27]

$$Cov(\phi) \geq (\mu F_Q)^{-1}. \tag{1}$$

For the estimation of two phases, when the input state is a pure state, the elements of QFIM are given by

$$F_{Q(ij)} = \langle\psi(\phi_1,\phi_2)|\frac{L_{\phi_i}L_{\phi_j}+L_{\phi_j}L_{\phi_i}}{2}|\psi(\phi_1,\phi_2)\rangle, i,j=1,2. \tag{2}$$

where $|\psi(\phi_1,\phi_2)\rangle$ is the output state after the input state undergoing an unitary phase-shifter operator and the symmetric logarithmic derivative L_{ϕ_i} is defined by [28]

$$L_{\phi_i} = 2\left[\frac{\partial|\psi(\phi_1,\phi_2)\rangle}{\partial\phi_i}\langle\psi(\phi_1,\phi_2)| + |\psi(\phi_1,\phi_2)\rangle\frac{\partial\langle\psi(\phi_1,\phi_2)|}{\partial\phi_i}\right]. \tag{3}$$

In the following, we consider single photons ($|1\rangle_1|1\rangle_2|1\rangle_3$) entering into each port of a three-mode nonlinear interferometer which can be realized by cascading two three-port beam splitters (tritters) and two nonlinear phase shifters. In this paper, we use the tritter in Ref. [29] which has been realized by femtosecond laser waveguide writing and it consists of three waveguides approaching three-dimensionally [30]. The unitary matrix U of the tritter is

$$U_T = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{\frac{2i\pi}{3}} & e^{-\frac{2i\pi}{3}} \\ 1 & e^{-\frac{2i\pi}{3}} & e^{\frac{2i\pi}{3}} \end{pmatrix}. \tag{4}$$