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Focusing of shifted vortex beams of arbitrary order with different polarization

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ABSTRACT

In this paper we study the focusing of shifted vortex beams of arbitrary order with different polarization. It is shown theoretically that the optical vortex of an arbitrary integer order m, displaced inside a certain axially symmetric beam, is equivalent to the finite sum of the on-axis vortices of orders from 0 to m inclusive. If the order of the shifted vortex beam is non-integer, the sum will be infinite. The fact that the misaligned higher-order vortex beams contain a set of lower-order vortex beams should be taken into account at a signal detecting. It is very actual for communication systems based on orbital angular moments (OAM) carrying laser beams division multiplexing, especially at the distortions and/or wandering of the vortex beams in a turbulent or random medium.

Pictures of focused shifted vortex beams, regardless of the order of the vortex and displacement, look approximately the same, as a crescent. To obtain another type of focal pictures, an optical vortex of the opposite sign can be used as an illuminating beam. For the numerical study of the considered effects simultaneously at several optical vortices of different orders, directions of rotation and polarization, a multi-order diffraction optical element was used in the conditions of sharp focusing. It is shown, that named characteristics significantly change the form of rotating intensity in the focal area. It may be useful for optical manipulations.

1. Introduction

Vortex beams are quite sensitive to various deviations in focusing optical systems — tilt, displacement from the optical axis, the presence of other aberrations. Note that the changes introduced by non-ideal optical systems, can be used in a positive aspect. In particular, the inclination of the incident beam or lens [1–4], the presence of astigmatism or ellipticity [5,6], as well as the use of cylindrical lenses [7] contributes to the vortex beam distortion, allowing to visualize its topological charge. Converters based on astigmatic transformations allow forming vortex beams from non-vortical distributions [8–10]. The presence of some other aberrations in the focusing system makes it possible to reduce the focal spot size in tight focusing of vortex beams [11–13]. Symmetry breaking in the optical system at the formation and focusing of vortex beams can lead to the "collapse" of high-order optical vortices into several first-order vortex components [14]. A similar effect is observed when the wavelength of the illuminating beam deviates from the base one, as well at presence of fabrication errors of the diffraction optical element [15,16]. The vortex beam of the first order in this case becomes asymmetric. As a rule, the order of the optical vortex instead of the integer becomes fractional [15,17,18].

Vortex beams displaced from the optical axis are also of interest in sharp focusing, since in this case a beam with a zigzag (rotating) trajectory of maximum intensity propagation is formed in the focal area [19]. Note that in [19] only the first-order vortex beam was considered.

Off-axis vortex beams of different orders may be conveniently generated by binary fork-shaped gratings, which are based on the interference of a vortex beam with an inclined plane beam (carrying spatial frequency) [20,21]. In this case, it is possible to simultaneously form several different beams propagating under the angles to the optical axis. Such multi-order (or multi-channel) optical elements are used for optical decomposition of laser fields on some basis, including angular harmonics [22–24].

In this paper we investigate theoretically and numerically properties of focused shifted vortex beams of arbitrary order with different polarization. Numerical simulations consider a multi-order diffraction optical

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element (DOE) consistent with several optical vortices of different orders under sharp focusing conditions in the Debye approximation [13]. In this case, it is possible to demonstrate the studied effects simultaneously on several optical vortices, including the opposite direction of rotation.

The obtained research results are relevant for multi-channel communication system based on OAM-carrying laser beams division multiplexing [25–28], especially considering distortions and/or walks of vortex beams in turbulent or random medium [29,30]. The results may also be useful for optical manipulations [31,32].

2. Theoretical analysis

The electric vector field in the focal region is described by the following equation:

$$\begin{aligned}\mathbf{E}(u,v,z) = &-\frac{if}{\lambda}\int_0^{\theta_{\max}}\int_0^{2\pi} B(\theta,\phi)T(\theta)\mathbf{P}(\theta,\phi)\times\\ &\times\exp\left[ik(x\sin\theta\cos\phi+y\sin\theta\sin\phi+z\cos\theta)\right]\times\\ &\times\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,,\end{aligned} \tag{1}$$

where (u,v,z) are the Cartesian coordinates in the focal region, (θ,ϕ) are spherical angular coordinates of the focusing system's output pupil, $\theta_{\max}$ is a maximum value of the azimuthal angle, associated with the numerical aperture (NA) of the system, $B(\theta,\phi)$ is the transmission function, $T(\theta)$ is the pupil's apodization function ($T(\theta)=\sqrt{\cos\theta}$ for aplanatic systems), $k=2\pi/\lambda$ is the wavenumber, λ is the wavelength of radiation, f is the focus length.

The vector of polarization transformation in the focal region for the Cartesian coordinates has the following form:

$$\begin{aligned}&\mathbf{P}(\theta,\phi)\\ &=\begin{bmatrix}1+\cos2\phi(\cos\theta-1) & \sin\phi\cos\phi(\cos\theta-1) & \cos\phi\sin\theta\\ \sin\phi\cos\phi(\cos\theta-1) & 1+\sin2\phi(\cos\theta-1) & \sin\phi\sin\theta\\ -\sin\theta\cos\phi & -\sin\theta\sin\phi & \cos\theta\end{bmatrix}\\ &\times\begin{pmatrix}c_x(\phi)\\ c_y(\phi)\\ c_z(\phi)\end{pmatrix}\end{aligned} \tag{2}$$

where $\mathbf{c}(\phi)=(c_x(\phi),c_y(\phi),c_z(\phi))^T$ is the polarization vector of the input field.

For the laser beams with a radial symmetry and mth order vortex phase:

$$B(\theta,\phi)=R(\theta)\exp(im\phi). \tag{3}$$

Then, Eq. (1) can be simplified:

$$\begin{aligned}&\mathbf{E}_m(\rho,\varphi,z)=-ikf\times\\ &\times\int_0^{\theta_{\max}} R(\theta)T(\theta)\mathbf{Q}_m(\rho,\varphi,\theta)\sin\theta\exp(ikz\cos\theta)\,\mathrm{d}\theta\,,\end{aligned} \tag{4}$$

where (ρ,φ,z) are the cylindrical coordinates in the focal region and components of the vector $\mathbf{Q}_m(\rho,\varphi,\theta)$ depend on the polarization of incident laser beam $\mathbf{c}(\phi)$ and represent a superposition of Bessel functions of various orders [13].

In this work we consider the input field in the form of off-axis optical vortices of arbitrary order within a given axisymmetric beam. First we write it in Cartesian coordinates:

$$\begin{aligned}&B(x,y)=\\ &=R\left(\sqrt{x^2+y^2}\right)\sum_p\left(\left(x-x_p\right)+i\left(y-y_p\right)\right)^{m_p}=\\ &=R\left(\sqrt{x^2+y^2}\right)\sum_p\left((x+iy)-\left(x_p+iy_p\right)\right)^{m_p}.\end{aligned} \tag{5}$$

For integer values of m_p, expression (5) can be converted using the binomial formula:

$$\begin{aligned}&B(x,y)=R\left(\sqrt{x^2+y^2}\right)\times\\ &\times\sum_p\left(\sum_{l=0}^{m_p}(-1)^l C_{m_p}^l\cdot(x+iy)^{m_p-l}\left(x_p+iy_p\right)^l\right).\end{aligned} \tag{6}$$

That is, the shifted vortex of order m_p is equivalent to the finite sum of the on-axis vortices of orders from 0 to m_p inclusive. So, from an algebraic point of view, we can use already known analytic expressions obtained for an on-axis vortex (4) if we ignore the bulkiness of the resulting expressions.

If m_p is not integer, the sum becomes infinite, and it will include on-axis optical vortices of all orders with the same sign.

Further it is more convenient to use polar coordinates:

$$\begin{aligned}B(r,\phi)&=R(r)\sum_p\left(\left(r\cos\phi-r_p\cos\phi_p\right)+\right.\\ &\quad\left.+i\left(r\sin\phi-r_p\sin\phi_p\right)\right)^{m_p}=\\ &=R(r)\sum_p\left(re^{i\phi}-r_pe^{i\phi_p}\right)^{m_p}=\\ &=R(r)\sum_p\left(\sum_{l=0}^{m_p}(-1)^l C_{m_p}^l\cdot\left(re^{i\phi}\right)^{m_p-l}\left(r_pe^{i\phi_p}\right)^l\right).\end{aligned} \tag{7}$$

For some values of m_p, we shall write the explicit expressions:

$$\begin{aligned}B_1(r,\phi)&=R(r)\left(re^{i\phi}-r_0e^{i\phi_0}\right);\\ B_2(r,\phi)&=R(r)\left(re^{i\phi}-r_0e^{i\phi_0}\right)^2=\\ &=R(r)\left[\left(re^{i\phi}\right)^2-2r_0e^{i\phi_0}re^{i\phi}+\left(r_0e^{i\phi_0}\right)^2\right];\\ B_3(r,\phi)&=R(r)\left(re^{i\phi}-r_0e^{i\phi_0}\right)^3=\\ &=R(r)\left[\left(re^{i\phi}\right)^3-3r_0e^{i\phi_0}\left(re^{i\phi}\right)^2+\right.\\ &\quad\left.+3\left(r_0e^{i\phi_0}\right)^2re^{i\phi}-\left(r_0e^{i\phi_0}\right)^3\right].\end{aligned} \tag{8}$$

As can be clearly seen, the superposition consists of vortex beams with the same direction of rotation. In this case, the pictures, regardless of the order of the vortex and displacement, will look about the same — as a "crescent" [19], reminding also the pictures for fractional orders [33]. This is expected, since the optical vortex of fractional order can be represented as an infinite series on integer orders [16].

To get pictures of another type, for example "camomile-shaped" [23], it is necessary to form a superposition of optical vortices with different directions of rotation. So, if we use a shifted vortex beam of the opposite sign as the illuminating beam, we shall get:

$$B(r,\phi)=\left(re^{-i\phi}-r_se^{-i\phi_s}\right)^{m_s}\sum_p\left(re^{i\phi}-r_pe^{i\phi_p}\right)^{m_p}. \tag{9}$$

In particular, for $m_s=1$ and $p=1$, $m_1=2$:

$$\begin{aligned}B(r,\phi)&=\left(re^{-i\phi}-r_se^{-i\phi_s}\right)\times\\ &\quad\times\left[\left(re^{i\phi}\right)^2-2r_0e^{i\phi_0}re^{i\phi}+\left(r_0e^{i\phi_0}\right)^2\right]=\\ &=\left[e^{i\phi}r^3-2r_0e^{i\phi_0}r^2+\left(r_0e^{i\phi_0}\right)^2e^{-i\phi}r\right]-\\ &\quad-r_se^{-i\phi_s}\left(re^{i\phi}-r_0e^{i\phi_0}\right)^2.\end{aligned} \tag{10}$$

The second term up to the coefficient is similar to the illumination by a plane beam. And the first term can be rewritten in the form of:

$$\begin{aligned}&\left[e^{i\phi}r^3-2r_0e^{i\phi_0}r^2+\left(r_0e^{i\phi_0}\right)^2e^{-i\phi}r\right]=\\ &=2\left(r_0e^{i\phi_0}\right)^2\cos(\phi)r+e^{i\phi}\times\\ &\quad\times\left[r^3-r\left(r_0e^{i\phi_0}\right)^2\right]-2r_0e^{i\phi_0}r^2.\end{aligned} \tag{11}$$

From (11) it is visible that the picture will contain "dumbbell". Obviously, for larger values of m_s the "camomiles" with different numbers of petals will be present.

3. Numerical simulation

This section presents the results of numerical simulation using expressions (1)–(2).

Fig. 1 shows the results of sharp focusing of a linearly polarized centered first-order vortex beam. Obviously, linear polarization introduces