



# Propagation properties of the rotating elliptical chirped Gaussian vortex beam in the oceanic turbulence

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## ABSTRACT

Based on the extended Huygens–Fresnel integral, the analytical formulas for the average intensity of the rotating elliptical chirped Gaussian vortex beam (RECGVB) in the oceanic turbulence are derived in this paper. Meanwhile, the evolutions of the intensity, the effective beam widths in the  $x$ - and  $y$ -directions and the dynamic center of gravity for the RECGVB propagating through the oceanic turbulence are explored by numerical examples. Results show that the intensity pattern of the RECGVB which presents two light spots at the beginning deforms and spins within a certain propagation range. Moreover, the position of the dynamic center of gravity can be modulated by the first-order chirped parameters while the profile of the intensity pattern can be affected by the second-order chirped parameters. In addition, the effects of the oceanic turbulence parameters and the initial beam width on the normalized intensity and the effective beam width of the RECGVB are discussed in detail.

## 1. Introduction

In recent years, the propagations of laser beams through the oceanic turbulence have become research hotspots [1,2] owing to their significant applications in underwater including the imaging system [3] and optical communication [4,5]. Up to now, a series of propagation characteristics have been achieved, including the scintillation index [6], spherical waves [7], the degree of polarization [8], average aperture [9] and the beam wander [10]. Furthermore, the properties of various optical beams propagating through the oceanic turbulence have been studied, such as the Gaussian-Schell model beams [11], the partially coherent model beam [12], the anti-specular Gaussian Schell-model beams [13], the asymmetrical optical beams [14] and so on. Additionally, the vortex beam which spreads with the helical wave front in a spiral way is paid for more and more attention in astrophysics [15], entanglement propagation [16], space singularity [17] and optical communications through turbulent channels [18–20].

On the other hand, the change of the carrier concentration in semiconductor lasers and amplifiers equips pulses with a frequency chirped factor which is one of the important factors used to manipulate pulses [21–23]. As for the spacial chirped factor, a linear chirped parameter introduces a transverse displacement of the beam at the phase transition point but not at the location of the point and a quadratic chirped parameter changes the phase transition point but not the beam profile [24]. Meanwhile, the method for setting microparticles into

rotation by using rotating beam has been widely applied to optical tweezers that can be used to manipulate and study cells [25,26]. Since the rotating beam does not rely on intrinsic properties of the particle, it can be applied to optical trapping [27]. Moreover, the properties of the rotating elliptical Gaussian beams have attracted quite a lot of interest through the nonlinear media [28], nonlocal media [29] and anisotropic media [30]. However, the investigation on the combination of the vortex, the chirped factors and the rotating elliptical Gaussian has not been reported yet. Here, we deeply study the propagation properties of the rotating elliptical chirped Gaussian vortex beam (RECGVB) in the oceanic turbulence, which has the potential to modulate the rotation of particles by changing the first-order and second-order chirped parameters and may be helpful for optical communication, imaging, and sensing in the underwater. Subsequently, the influences of the chirped parameters, the initial beam width and the oceanic turbulence parameters on the normalized intensity, the dynamic center of gravity and the effective beam width are illustrated in detail.

This paper is organized as follows. Firstly, we obtain the intensity expression of the RECGVB propagating in the oceanic turbulence in Section 2. In Section 3, we take the chirped parameters, the propagation distance, the oceanic turbulence parameters as well as the initial beam width into consideration to study the evolutions of the normalized intensity, the effective beam width and the dynamic center of gravity in detail. At last, the results of this paper are summarized in Section 4.

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## 2. The analytical expressions of the RECGVB in the oceanic turbulence

In the Cartesian coordinate system, the  $z$ -axis is regarded as the propagation axis. The electric field of the RECGVB in the original plane  $z = 0$  is written as

$$E(\mathbf{r}_0, 0) = A_0 \exp \left( i\beta_{1x} \frac{x_0}{aw} + i\beta_{1y} \frac{y_0}{bw} + i\beta_{2x} \frac{x_0^2}{a^2w^2} + i\beta_{2y} \frac{y_0^2}{b^2w^2} \right) \times \exp \left( -\frac{x_0^2}{a^2w^2} - \frac{y_0^2}{b^2w^2} - i \frac{x_0y_0}{c^2w^2} \right) \left( \frac{x_0}{w} + \frac{iy_0}{w} \right), \quad (1)$$

where  $\mathbf{r}_0 = (x_0, y_0)$  represents the initial position of the electric field distribution;  $a$ ,  $b$  and  $c$  are the elliptical parameters;  $w$  is the initial beam width;  $\beta_{1x}$  and  $\beta_{1y}$  are the first-order chirped parameters while  $\beta_{2x}$  and  $\beta_{2y}$  are the second-order chirped parameters. Based on the extended Huygens–Fresnel integral, the average intensity of the RECGVB propagating through the oceanic turbulence under the paraxial approximation can be expressed as

$$\langle I(\mathbf{r}, z) \rangle = \frac{k^2 A_0^2}{4\pi^2 z^2} \iiint_{-\infty}^{+\infty} E(\mathbf{r}_{01}, 0) E^*(\mathbf{r}_{02}, 0) \exp \left[ -\frac{ik}{2z} (\mathbf{r}_{01} - \mathbf{r})^2 + \frac{ik}{2z} (\mathbf{r}_{02} - \mathbf{r})^2 \right] \langle \exp [\Psi(\mathbf{r}_{01} - \mathbf{r}) + \Psi^*(\mathbf{r}_{02} - \mathbf{r})] \rangle d^2\mathbf{r}_{01} d^2\mathbf{r}_{02}, \quad (2)$$

where  $\mathbf{r}_{01} = (x_1, y_1)$  and  $\mathbf{r}_{02} = (x_2, y_2)$  denote the initial position vectors on the  $z$  plane;  $\mathbf{r}$  is defined as an arbitrary position vector on the  $z$  plane;  $k$  is the wave number with  $k = \frac{2\pi}{\lambda}$ ;  $\lambda$  is the wavelength of the RECGVB;  $*$  and  $\langle \cdot \rangle$  stand for the complex conjugate and the ensemble average, respectively;  $\Psi(\mathbf{r}_{01} - \mathbf{r})$  and  $\Psi(\mathbf{r}_{02} - \mathbf{r})$  indicate the complex phase perturbations induced by the oceanic turbulence. And the ensemble average of the turbulent ocean is stated as [31]

$$\langle \exp [\Psi(\mathbf{r}_{01} - \mathbf{r}) + \Psi^*(\mathbf{r}_{02} - \mathbf{r})] \rangle = \exp \left\{ -4\pi^2 k^2 z \int_0^{+\infty} \int_0^1 \kappa \Phi_n(\kappa) [1 - J_0(\kappa|(1 - \xi)(\mathbf{r}_{02} - \mathbf{r}_{01})|)] d\xi d\kappa \right\}, \quad (3)$$

where  $J_0$  is the 0th order Bessel function,  $\Phi_n(\kappa)$  is the spatial power spectrum of the refractive-fluctuations of the oceanic turbulence. Since the fluctuation of the source is much larger than that of the random medium, it is possible to approximate the Bessel function by its two first terms i.e.  $J_0 = 1 - \frac{1}{4}\kappa^2$  [32] and Eq. (3) can be simplified as

$$\begin{aligned} & \exp \left\{ -4\pi^2 k^2 z \int_0^{+\infty} \int_0^1 \kappa \Phi_n(\kappa) [1 - J_0(\kappa|(1 - \xi)(\mathbf{r}_{02} - \mathbf{r}_{01})|)] d\xi d\kappa \right\} \\ &= \exp \left\{ -\frac{\pi^2 k^2 z}{3} \int_0^{+\infty} \kappa^3 \Phi_n(\kappa) d\kappa [(\mathbf{r}_{02} - \mathbf{r}_{01})^2] \right\} \\ &= \exp \left\{ -T(\mathbf{r}_{02} - \mathbf{r}_{01})^2 \right\}, \end{aligned} \quad (4)$$

with

$$T = \frac{\pi^2 k^2 z}{3} \int_0^{+\infty} \kappa^3 \Phi_n(\kappa) d\kappa. \quad (5)$$

Considering the effects of temperature and salinity fluctuations on the oceanic turbulence, the spatial power spectrum of the oceanic turbulence model is given by [1]

$$\Phi_n(\kappa) = 0.388 \times 10^{-8} \epsilon^{-\frac{1}{3}} \kappa^{-\frac{11}{3}} \frac{\chi_t}{\zeta^2} \left[ 1 + 2.35(\kappa\eta)^{\frac{2}{3}} \right] \times (\zeta^2 e^{-A_T \delta} + e^{-A_S \delta} - 2\zeta e^{-A_{TS} \delta}). \quad (6)$$

Substituting Eq. (6) into Eq. (5), we can obtain

$$T = \frac{\pi^2 k^2 z}{3} \left\{ \int_0^{+\infty} 0.388 \times 10^{-8} \epsilon^{-\frac{1}{3}} \kappa^{-\frac{2}{3}} \frac{\chi_t}{\zeta^2} \left[ 1 + 2.35(\kappa\eta)^{\frac{2}{3}} \right] \times (\zeta^2 e^{-A_T \delta} + e^{-A_S \delta} - 2\zeta e^{-A_{TS} \delta}) d\kappa \right\}, \quad (7)$$

where  $\delta = 8.284(\kappa\eta)^{4/3} + 12.978(\kappa\eta)^2$  [32];  $\eta = 10^{-3}$  is the Kolmogorov micro scale (inner scale);  $\epsilon$  is the dissipation rate of turbulent kinetic energy per unit mass of fluid ranging from  $10^{-10} \text{ m}^2/\text{s}^3$  to  $10^{-1} \text{ m}^2/\text{s}^3$  [33];  $\chi_t$  is the rate of dissipation of average-square temperature taking values in the range from  $10^{-10} \text{ K}^2/\text{s}$  to  $10^{-4} \text{ K}^2/\text{s}$  [34] ( $K$  is the degree Kelvin);  $\zeta$  is the relative strength of temperature and salinity fluctuations, which ranges from  $-5$  to  $0$  in the oceanic turbulence. After substituting  $A_T = 1.863 \times 10^{-2}$ ,  $A_S = 1.9 \times 10^{-4}$  and  $A_{TS} = 9.41 \times 10^{-3}$  into Eq. (7), we can simplify it as

$$T = 1.293 \times 10^{-9} \pi^2 k^2 z \zeta^{-2} \epsilon^{-\frac{1}{3}} \eta^{-\frac{1}{3}} \chi_t (47.5708 - 17.6710\zeta + 6.78335\zeta^2). \quad (8)$$

Afterwards, substituting Eqs. (4) and (8) into Eq. (2), we obtain the analytic expression of the average intensity for the RECGVB as

$$\begin{aligned} I_o &= \frac{k^2 A_0^2}{\sqrt{p_1 4z^2 w^2}} \exp \left( \frac{M^2}{4p_1} \right) \left\{ \frac{1}{2p_1 \sqrt{A_1' E_1 E_1'}} \exp \left( \frac{B_1'^2}{4A_1'^2} + \frac{G_1'}{E_1'} + \frac{F_1'^2}{E_1'} \right) \right. \\ &\times \left[ \frac{A_1}{2E_1} + \frac{A_1 F_1^2}{E_1^2} + \frac{B_1 F_1}{E_1} + \frac{A_1' G_1^2}{E_1'^2} + \frac{2B_1' G_1' + A_1'}{2E_1'} \right] \\ &+ \frac{1}{A_2'' \sqrt{A_2'' E_2 E_2'}} \exp \left( \frac{B_2''^2}{A_2''^2} + \frac{F_2^2}{E_2^2} + \frac{F_2'^2}{E_2^2} \right) \\ &\times \left[ \frac{A_2}{2E_2} + \frac{A_2 F_2^2}{E_2^2} + \frac{B_2 F_2}{E_2} + \frac{2F_2^2 C_0}{E_2'^2} + \frac{B_2' F_2' i + C_0}{E_2'} \right] \left. \right\}, \end{aligned} \quad (9)$$

with  $C_0 = \frac{T}{4p_1 c^2 w^2}$ ,  $M = \frac{i\beta_{1x}}{aw} + \frac{ikx}{z}$ ,  $N = \frac{i\beta_{1y}}{bw} + \frac{iky}{z}$ ,  $p_1 = \frac{1}{a^2 w^2} - \frac{i\beta_{2x}}{a^2 w^2} + \frac{ik}{2z} + T$ ,  $p_2 = \frac{1}{b^2 w^2} - \frac{i\beta_{2y}}{b^2 w^2} + \frac{ik}{2z} + T$ ,  $p_3 = \frac{1}{a^2 w^2} + \frac{i\beta_{2x}}{a^2 w^2} - \frac{ik}{2z} + T$ ,  $p_4 = \frac{1}{b^2 w^2} + \frac{i\beta_{2y}}{b^2 w^2} - \frac{ik}{2z} + T$ ,  $A_1' = 2T - \frac{2C_0}{A_1' c^2 w^2}$ ,  $B_1' = -\frac{B_1' i}{2A_1' c^2 w^2} + M$ ,  $C_1' = -\frac{4C_0 p_1 i}{A_1' c^2 w^2} - 2Ti + \frac{2C_0 i}{A_1' c^2 w^2}$ ,  $E_1' = p_3 - \frac{T^2}{p_1} + \frac{4C_0^2}{A_1'^2}$ ,  $A_1'' = p_2 + \frac{1}{4p_1 c^4 w^4}$ ,  $B_1'' = N - \frac{Mi}{2p_1 c^2 w^2}$ ,  $A_1 = \frac{A_1' F_1'^2}{E_1'^2} + \frac{C_1' F_1'}{E_1'} - \frac{4C_0 p_1}{A_1' c^2 w^2}$ ,  $B_1 = \frac{2A_1' F_1' G_1'}{E_1'^2} + \frac{C_1' G_1' + B_1' F_1'}{E_1'} - \frac{B_1''}{2A_1' c^2 w^2} - Mi$ ,  $F_1' = -\frac{2C_0 T i}{A_1' c^2 w^2} + \frac{i}{2c^2 w^2}$ ,  $G_1' = \frac{MT}{2p_1} - \frac{M}{2} - \frac{B_1' C_0 i}{A_1' c^2 w^2}$ ,  $E_1 = p_4 - \frac{T^2}{A_1'} - \frac{F_1'^2}{E_1'}$ ,  $F_1 = \frac{T B_1''}{2A_1'} - \frac{N}{2} + \frac{F_1' G_1'}{E_1'}$ ,  $A_2' = -2C_0 i + Ti$ ,  $A_2'' = p_2 + \frac{1}{4p_1 c^4 w^4}$ ,  $B_2'' = -\frac{C_0 Mi}{T} + \frac{N}{2}$ ,  $A_2 = T + \frac{2G_2'^2 C_0}{E_2'^2} + \frac{A_2' G_2'}{E_2'}$ ,  $B_2 = B_2'' + \frac{A_2' F_2^2}{E_2^2} + \frac{B_2'' G_2' i}{E_2^2} + \frac{4F_2^2 G_2' C_0}{E_2'^2}$ ,  $E_2' = \frac{4C_0^2}{A_2'^2} + p_3 - \frac{T^2}{p_1}$ ,  $F_2' = \frac{TM}{2p_1} - \frac{M}{2} - \frac{2B_2'' C_0 i}{A_2'^2}$ ,  $G_2' = -\frac{2TC_0 i}{A_2'^2} + \frac{i}{2c^2 w^2}$ ,  $E_2 = p_4 - \frac{T^2}{A_2''} - \frac{G_2'^2}{E_2^2}$ ,  $F_2 = \frac{T B_2''}{A_2''} - \frac{N}{2} + \frac{F_2' G_2'}{E_2^2}$ .

In addition, the diffusion feature of the RECGVB is characterized by the effective beam width of  $j$ -direction in the  $z$  plane, which is defined as [35]

$$W_{jz} = \sqrt{\frac{2 \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} j^2 \langle I_o(\mathbf{r}, z) \rangle dx dy}{\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \langle I_o(\mathbf{r}, z) \rangle dx dy}}. \quad (j = x, y) \quad (10)$$

The movement of the dynamic center of gravity can be derived from the average intensity of the beam, which is defined as [36]

$$X_c = \frac{\int_{-\infty}^{+\infty} x I_o dx dy}{\int_{-\infty}^{+\infty} I_o dx dy}. \quad (11)$$

## 3. Numerical discussions and results

With the analytical expression of the RECGVB propagating in the oceanic turbulence, we further investigate the propagation properties of the RECGVB by illustrating the examples including the intensity, the dynamic center of gravity and the effective beam width. Unless specified in captions, some parameters are selected as follows:  $a = 2$ ,  $b = 0.5$ ,  $c = 1.2$ ,  $w = 2 \text{ mm}$ ,  $A_0 = 1$ ,  $\lambda = 0.55 \text{ }\mu\text{m}$ ,  $\epsilon = 10^{-7} \text{ m}^2/\text{s}^3$ ,  $\zeta = -2.5$  and  $\chi_t = 10^{-9} \text{ K}^2/\text{s}$ .

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