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# Wavefront reconstruction based on the Fresnel zone plane with hybrid focal lengths

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ABSTRACT

Phase retrieval using single farfield image data is the key to many applications. However, this method easily falls into stagnation because of the multiple solutions. A wavefront reconstruction algorithm based on the hybrid focal lengths Fresnel zone plane is proposed in this paper. By using the proposed method the partially focused and defocused information of wavefront can be captured on just one farfield image. This method can effectively solve the phase ambigiuty, and the optical structure is very simple and easy to implement. Simulation results show that the proposed method can reconstruct wavefront aberration very precisely with just one far field.

## 1. Introduction

Phase retrieval (PR) is an iterative process of wavefront recovery from a known intensity distribution. Owing to the high detection accuracy and low environmental requirement, PR has become an attractive candidate to the wavefront sensor (WFS) in adaptive optics [1,2]. Because of the development of computer speed, PR will have a high potential in active optical systems [3].

Gerchberg and Saxton proposed the GS algorithm to recover the wavefront aberration [4]. This algorithm can achieve typical convergence, but its iterative process easily falls into stagnation. For instance, the true pupil field $f(x, y)$ and its twin $f^*(-x, -y)$ have the same Fourier modulus; therefore, the algorithm tries to recover both together but fails to provide a solution [5]. Shuai Wang proposed a method by analysing the farfield image relevancy of the Zernike polynomial with positive and negative coefficients to reveal the uniqueness of the solution of the GS algorithm [6].

To overcome the two-fold ambiguity of the GS algorithm, Gonsalves proposed the phase diversity (PD) algorithm in 1979 [7]. Currently, PD has been successfully applied in several practical systems [8–10]. However, this method introduces a defocused plane to increase the constrained intensity information for the reconstruction of wavefront, which will sacrifice the simplicity of optical structure. Löfdahl proposed a PD sensor using a beam splitter to capture the focused and defocused images in one CCD [11]. For the same purpose a PD sensor with a distorted diffraction grating was presented by Blanchard in 2000 [12]. Nevertheless, a more complex structure, interaction of highfrequency information and the dynamic range of CCD limit the practical application of these methods in PD.

Min Li and Xinyang Li proposed a method based on the linear phase retrieval (LPR) to reconstruct small aberrations from a single farfield image [13,14]. In 2010, Serge Meimon proposed the linearised focal plane technique, which is effective for lowerorder aberrations. To solve the above problem of LPR, Bing Dong et al. demonstrated a hybrid PR algorithm using a combination of LPR and GS in 2015 [15]. The estimation result of LPR is used as a prior knowledge to the speed convergence of GS. In this way, the higherorder aberrations are basically recovered, but the problem of multiple solutions remains unsolved.

In 2016, Alexandra Z. Greenbaum proposed another algorithm by solving the sign ambiguity of the phase to reconstruct a wavefront with a single far field A nonredundant mask (NRM) that consists of a set of holes is used in the PR algorithm. This method uses a pair of in-focus

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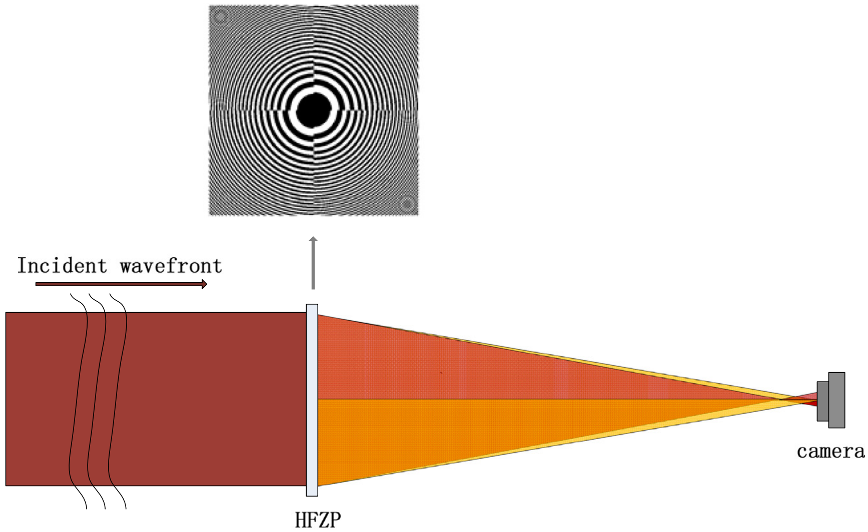


**Fig. 1.** Optical arrangement of the wavefront reconstruction method based on the HFZP.

**Table 1**
Relationship between the iteration speed and the focal length.

| | $f_{13}=250$ $f_{24}=300$ | $f_{13}=500$ $f_{24}=600$ | $f_{13}=1000$ $f_{24}=1200$ | $f_{13}=1500$ $f_{24}=1800$ |
|---|---|---|---|---|
| $N$ | 75 | 50 | 34 | 32 |

images, one acquired with NRM in the pupil and the other acquired with the full pupil. After performing hundreds of iterations the NRM constraint is removed, and the typical GS algorithm is used to obtain the true aberration [16]. This is an excellent approach; however, the precision of the NRM cutting-in optical path and the complexity of the device negatively affect the application of this algorithm in PR.

It is well known that the Fresnel zone plane (FZP) is an ideal binary optical imager [17]. According to the binary diffractive element fabrication flexibility, a wavefront reconstruction algorithm based on the hybrid focal lengths Fresnel zone plane (HFZP) is proposed in this paper. It is attractive that the partially focused and defocused information of wavefront can be captured on just one farfield image. This proposed method can efficiently solve the retrieval problem using a single far field.

In this paper, the feasibility and the uniqueness of the solution of this method are analysed. Furthermore, this method is also compared with the GS algorithm.

This paper is organised as follows: The basic principle of the HFZP algorithm is introduced in Section 2 to deduce some equations about this hybrid focal lengths method. A numerical simulation is performed to compare the reconstruction performance of the HFZP algorithm with that of the traditional GS algorithm in Section 3. The influence of noise on the HFZP reconstruction performance is analysed in Section 4. A summary of this work is presented in Section 5.

## 2. Principle of the hybrid focal lengths retrieval method

We propose a wavefront reconstruction method based on the HFZP. It has a very concise optical arrangement as shown below. The incident light is modulated by the HFZP and then propagated to the far field where a camera is located, as shown in Fig. 1. The HFZP is made up of four parts with different focal lengths.

We set the focal length of the first and third quadrants of the HFZP to $f_{13}$ and the focal length of the second and fourth quadrants to $f_{24}$. According to the FZP theory, the boundary radius of the $n$th zone is given as follows [18]:

$$\begin{aligned} r_{13\_n} &= \sqrt{n\lambda f_{13}} \\ r_{24\_n} &= \sqrt{n\lambda f_{24}} \end{aligned} \tag{1}$$

where $\lambda$ is the wavelength $r_{13\_n}$ denotes the radius of the $n$th zone of the first and third quadrants, and $r_{24\_n}$ represents the radius of the $n$th zone of the second and fourth quadrants

In our design, the HFZP is based on the phase FZP, where the even rings of the FZP have a $\pi$ phase-shift with respect to the odd rings The modulation phase of the first and third quadrants of the HFZP is written as follows:

$$\Delta\varphi(x_1,y_1,0)\begin{cases} = 0, & (r_{13\_2i} \le \sqrt{{x_1}^2+{y_1}^2} < r_{13\_2i+1},\ i=0,1,2,3\cdots) \\ = \pi, & (r_{13\_2i-1} \le \sqrt{{x_1}^2+{y_1}^2} < r_{13\_2i},\ i=1,2,3\cdots) \end{cases} \tag{2}$$

The modulation phase of the second and fourth quadrants of the HFZL is written as follows:

$$\Delta\varphi(x_1,y_1,0)\begin{cases} = 0, & (r_{24\_2i} \le \sqrt{{x_1}^2+{y_1}^2} < r_{24\_2i+1},\ i=0,1,2,3\cdots) \\ = \pi, & (r_{24\_2i-1} \le \sqrt{{x_1}^2+{y_1}^2} < r_{24\_2i},\ i=1,2,3\cdots) \end{cases} \tag{3}$$

where $x_1$ and $y_1$ are the position coordinates of the modulated plane. The modulated field can be expressed as follows:

$$E\left(x_1,y_1,0\right) = E_{near}\left(x_1,y_1,0\right)\cdot \exp(i\Delta\varphi_{(x_1,y_1,0)}) \tag{4}$$

where $E_{near}\left(x_1,y_1,0\right)$ is the input light field.

By using the Fresnel diffraction based algorithm [19,20], when the distance of image plane is $f_{13}$, the far field$E_{far}(x,y,f_{13})$ is given as follows:

$$\begin{aligned} E_{far}(x,y,f_{13}) = {}& \frac{\exp(ikf_{13})}{i\lambda f_{13}} \exp\left[\frac{ik}{2f_{13}}(x^2+y^2)\right] \\ & \times F\left\{E(x_1,y_1,0)\exp\left[\frac{ik}{2f_{13}}({x_1}^2+{y_1}^2)\right]\right\} \end{aligned} \tag{5}$$

where $k=2\pi/\lambda$ is the wave number and $F$ is the Fourier transform.

The symmetrical wavefront has the typical sign ambiguity [5]. Therefore, we suppose that the inputted wavefront is a defocused aberration, as shown in Fig. 2(a). This incident light is transmitted to the HFZP. From the above formula, the farfield image is obtained, as illustrated in Fig. 2(c). When the inputted wavefront has a negative