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Free electron laser oscillator efficiency

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ABSTRACT

The efficiency of Free Electron Laser (FEL) Oscillator devices is a fairly complicated function of the various parameters which characterize the device itself. We explore the relevant dependences by the use of the scaling formulas describing the FEL Oscillator dynamics and providing the relevant design elements. We obtain a quantitative dependence of the efficiency on the key FEL parameters (small signal gain coefficient, saturation intensity, electron beam qualities...) and discuss the use of the results obtained in the paper as a further element for the FEL Oscillator optimization.

1. Introduction

The efficiency of Free Electron Laser (FEL) devices determines the amount of power which is transferred from the electron to the optical beam [1]. The problems associated with the relevant optimization have been one of the pivotal point of their design strategy, since the early proposals. The possibility of enhancing the output power by the use of undulators with non-constant parameters had been considered since the very beginning of the study of FEL amplifiers [2]. Tapered undulators have provided significant increase of the efficiency in devices operating at long wave-lengths [3], but, regarding X-ray SASE regime, the efficiency is limited to few per-mills by the values of the Pierce parameter ρ and by an insufficient gain guiding [4].

In the case of FEL devices in the oscillator regime (FELo) the reference parameter, ruling the power exchange between electrons and radiation, is associated with the inverse of the number of undulator periods. It must however be stressed that the inclusion of the cavity losses (active and passive) is a further element to be accounted for in the design optimization, through an appropriate definition of the extraction efficiency [5,6]. The problem is always under active consideration and,in a more recent times, significant effort towards the realization of high efficiency FELs has been accomplished within the framework of the TESSA/O proposal [7,8] in which a scheme of a high efficiency FEL oscillator, based on the application of the tapering enhanced stimulated superradiant amplification scheme (hence the acronym) has been proposed. Very roughly, talking about FEL oscillators, we can split the efficiency optimization in two parts. The first concerns the power transfer from electron to radiation, which may occur through tapered undulators. The second is associated with the extraction of the intracavity equilibrium power from the resonator.

In this paper we deal with extraction efficiency optimization of a constant parameter FEL oscillator, within the framework of a semi-analytical model. We use theoretical concepts emerged during the eighties and nineties of the last century [9–17] to elaborate a procedure yielding design including in an easily computational scheme all the physical quantities allowing the design and the optimization of a FEL oscillator. The starting point of our analysis is adapting to the FEL cases a procedure largely exploited in conventional laser physics [18]. The goal of the paper is that of obtaining the optimal extraction conditions using semi-analytical formulas providing the extraction efficiency in terms of the parameters characterizing the operation of a FELo.

Before entering the specific details of FELo optimization, we would like to highlight and recall that concepts analogous to those we are going to use in the rest of the paper have been already developed for the laser power output, relative to a lightly coupled laser oscillator [18], i.e. to an oscillator whose mirrors have a reflectivity slightly less than one. It can be shown [18] that the total output intensity in the steady-state oscillation condition can be expressed as:

$$I_{out} = \delta_e \left[\frac{2\alpha_{m0} p_m}{\delta_0 + \delta_e} - 1 \right] \frac{I_S}{2} \quad (1)$$

where $2\alpha_{m0} p_m$ is the unsaturated round-trip laser gain in the medium, $2\alpha_{m0}$ being the unsaturated gain and p_m two times the medium length, δ_0 is the internal cavity loss and δ_e is the output coupling, i.e. the total fractional power coupled out per round trip through the mirrors. We

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Table 1
Reference parameters for the design of the FEL devices. We have denoted by B_0 the amplitude of the undulator magnetic field, e and m_e are respectively the electron charge and mass, c is the speed of light, γ is the relativistic Lorentz factor and J_0, J_1 are the Bessel function of order 0 and 1.

N	Number of und. periods
λ_u	Undulator period
$L = N\lambda_u$	Undulator length
$\lambda_0 = \frac{1}{2\gamma^2}\left(1+K^{*2}\right)$	Resonance wavelength
J	Electron beam current density
I_A	Alfven current
$K = eB_0\lambda_u/2\pi m_e c$	Undulator parameter
$K^* = K/\sqrt{2}$	For helical undulator
$K^* = K$	For planar undulator
$\xi = \frac{1}{2}K^{*2}\left(1+K^{*2}\right)^{-1}$	
$f_b = 1$	Bessel factor for helical undulator
$f_b = J_0(\xi) - J_1(\xi)$	Bessel factor for planar undulator

have denoted by I_S the saturation intensity of an atomic or molecular laser. The optimum coupling coefficient is obtained by differentiating Eq. (1) with respect to δ_e:

$$\delta_{e,opt} = \sqrt{2\alpha_{m0}p_m\delta_0} - \delta_0. \tag{2}$$

The output intensity obtained in correspondence of the optimum output coupling is therefore:

$$I_{out,opt} = \left(1 - \sqrt{\frac{\delta_0}{2\alpha_{m0}p_m}}\right)^2 \alpha_{m0}p_m I_S. \tag{3}$$

It is possible to recognize in the term $\alpha_{m0}p_m I_S$ the available intensity I_{avail} from the laser medium, whence the extraction efficiency can be defined as $E_{ex} = I_{out}/I_{avail}$:

$$E_{ex} = \left(\frac{\delta_e}{\delta_0+\delta_e} - \frac{\delta_e}{2\alpha_{m0}p_m}\right). \tag{4}$$

The maximum value for the extraction efficiency is obtained for $\delta_e = \delta_{e,opt}$:

$$E^*_{ex} = \left(1 - \sqrt{\frac{\delta_0}{2\delta_{m0}p_m}}\right)^2. \tag{5}$$

Eq. (5) is extremely important in the sense that even for very small internal cavity losses the extraction efficiency can be significantly reduced.

2. FEL oscillator efficiency

The intracavity dynamics of FEL oscillators is described in terms of the pivotal parameters, small-signal gain coefficient g_0 and saturation intensity I_S [19], which are implicitly contained in the definition of the Colson's dimensionless amplitude and current [1].

The phenomenological role of these parameters is the same as for the ordinary lasers and the saturation intensity is defined as the intracavity power density that halves the small-signal (unsaturated) gain. In the following we will use the notation of FEL devices and specify small-signal and saturation intensity as

$$\begin{aligned} g_0 &= \frac{16\pi}{\gamma}\frac{|J|}{I_A}\lambda_0 L N^2 \xi f_b^2 \\ I_S &= \frac{c}{8\pi}\left(\frac{m_e c^2}{e}\right)^2\left(\frac{\gamma}{N}\right)^4\left(\lambda_u K^* f_b\right)^{-2}. \end{aligned} \tag{6}$$

Their importance stems from the fact that they merge most of the quantities specifying the design parameters of the device itself, which are summarized in Table 1 for later convenience.

The following identity, where P_E is the e-beam Power density, links small signal gain coefficient g_0 and saturation intensity I_s

$$\begin{aligned} g_0 I_S &= \frac{1}{2N}P_E \\ P_E &= E\cdot\frac{|J|}{e} = \frac{m_e c^2}{e}\gamma\,|J| = 2N g_0 I_S \end{aligned} \tag{7}$$

and stress the phenomenological importance of these two parameters. Their product yields the maximum power transferred from the electrons to the laser, in the case of a FEL (be it an amplifier or an oscillator) operating with a constant parameter undulator.

The problem is however more complicated than it may appear, because Eq. (7) does not include effects due to high gain or inhomogeneous broadening effects, which will be discussed afterwards. We remind therefore that the maximum small-signal gain is a non-linear function of g_0 which can be approximated as [20]

$$\begin{aligned} G_M &\cong g_0 f(g_0) \\ f(g_0) &= 0.85 + 0.192 g_0 + 4.23\cdot 10^{-3} g_0^2 \end{aligned} \tag{8}$$

and that an analogous correction affects the saturation intensity, whose dependence on the small-signal coefficient is expressed by the formula

$$\begin{aligned} I_S(g_0) &= \frac{1.078\cdot I_S}{P\left(g_0\right)} \\ P\left(g_0\right) &= 1 + 0.19\,g_0 - 8.7\cdot 10^{-3} g_0^2 + 2.7\cdot 10^{-4} g_0^3. \end{aligned} \tag{9}$$

It should be noted that even though the following identity holds

$$\frac{G_M I_S(g_0)}{0.85\,P_E} \cong \frac{1}{2N}, \tag{10}$$

which states that the product of gain and saturation intensity is almost insensitive to the gain coefficient, we cannot draw the conclusion that the FEL oscillator efficiency is just provided by the inverse of the number of undulator periods.

The identity in Eq. (7) is barely correct for an amplifier but to provide that of an oscillator, we should proceed through the following steps

1. Define the equilibrium intracavity intensity by means of the cavity losses of the device, namely [20]

$$I_e = \left(\sqrt{2}+1\right)\left(\sqrt{\frac{1-\eta}{\eta}G_M} - 1\right) I_S. \tag{11}$$

2. Evaluate the output power by introducing the extraction losses .

The last point will be considered later and for the moment we define the intracavity equilibrium power efficiency E_f, whose derivation proceeds as it follows[1]

$$\begin{aligned} E_f(g_0,\eta) &\cong \frac{\chi(g_0,\eta)}{2N}, \\ E_f(g_0,\eta) &= \frac{I_e}{P_E} \\ \chi(g_0,\eta) &= \left(\sqrt{2}+1\right)\left(\sqrt{\frac{1-\eta}{\eta}\frac{f(g_0)}{g_0}} - \frac{1}{g_0}\right)\frac{1.078}{P(g_0)}. \end{aligned} \tag{12}$$

In the previous equation the function $\chi(g_0,\eta)$ yields the FEL Oscillator intracavity efficiency with the inclusion of high-gain corrections from Eqs. (8), (9) and by using the identity (7).

The behavior of $\chi(g_0,\eta)$ vs. g_0 for different values of the losses is given in Fig. 1, which indicates that an optimum values of the small signal coefficient exists, which in turns defines an optimum current.

The condition $\chi(g_0,\eta) = 0$ defines the threshold gain coefficient $g_0^{th}(\eta)$, which is a function of the cavity losses. In correspondence of such a value we can define the so called starting current density as

$$J^{th}(\eta) = \frac{\gamma}{16\pi}\frac{1}{N^2\xi f_b^2}\frac{I_A}{\lambda_0 L}g_0^{th}(\eta). \tag{13}$$

The analytical expression for $g_0^{th}(\eta)$ obtained by means of the Cardano's formula [21] may be approximated as

$$g_0^{th}(\eta) \cong 2.214\left(\sqrt{1+1.063\frac{\eta}{1-\eta}} - 1\right). \tag{14}$$

The behavior of $g_0^{th}(\eta)$ is plotted in Fig. 2.

[1] In the following for reason of continuity we will denote the losses by η and not with δ as is from Eqs. (1) to (5).