Optics Communications 425 (2018) 71–79

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Optical propagation through anisotropic metamaterials: Application to metallo-dielectric stacks

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ARTICLE INFO

Keywords:
Anisotropic metamaterial
Hyperbolic metamaterial
Berreman matrix method
Transfer matrix method
Effective medium

ABSTRACT

We perform numerical simulations to compare the Berreman matrix method using effective medium results for an anisotropic material with exact calculations of a multi-layer metallo-dielectric stack using the transfer matrix method and finite element techniques. Results are given for a wide band of wavelengths and incident angles. For fixed sample thickness the number of layers is increased to study convergence of the optical characteristics (transmittance and reflectance). It is shown that the Berreman matrix method with effective medium results for an anisotropic material provides a fast and reliable estimate of the optical characteristics of the composite material. The Berreman technique readily leads to the transfer function matrix for propagation in anisotropic materials.

1. Introduction

Metamaterials are widely known in the field of optics because of their unique electromagnetic (EM) properties [1–3]. Metamaterials are artificially engineered structures designed to interact with EM radiation to achieve exotic material properties such as negative permittivity, negative permeability, negative refractive index, etc. leading to applications such as perfect imaging, optical filters, and coatings for special applications [2]. Such materials can be constructed, for instance, in the form a multilayered metallo-dielectric (MD) structure comprising alternating layers of metal and dielectric which can be modeled as a bulk anisotropic medium using effective medium theory [1]. These anisotropic metamaterials are believed to display interesting properties, including negative refraction and super-resolution in the near and/or far-field [1].

The propagation of EM waves in a medium is determined by its electric permittivity, ϵ and magnetic permeability, μ [3,4]. In an anisotropic metamaterial ϵ and μ are tensor quantities [5,6]. The anisotropy in these materials can be expressed as a diagonal matrix of ϵ and μ with their principal components having different values [6,7]. Hyperbolic metamaterials (HMMs) are a form of anisotropic material where the dielectric tensor elements have opposite signs [7–9]. There are two types of HMMs which can be distinguished by the signs of the principal elements of the diagonal permittivity matrix. Important dispersion characteristics of the hyperboloid are determined by whether the medium dielectric tensor principal components satisfy $\epsilon_{zz} < 0; \epsilon_{xx}, \epsilon_{yy} > 0$ or $\epsilon_{xx}, \epsilon_{yy} < 0; \epsilon_{zz} > 0$ [10]. Negative refraction can be achieved through the hyperbolic dispersion of these materials [11,12].

Anisotropic metamaterials are fabricated as a stack comprising alternating layers of metal and dielectric films, often referred to as metallo-dielectrics [2,6]. As shown by Argyropoulos et al. [1] for a hypothetical case, and using effective medium theory, the MD stack can be represented as a homogeneous anisotropic bulk material where the permittivities along the principal diagonal can have opposite signs owing to the negative (real part of the) permittivity of the metal. Conceptually, the reason for the anisotropy along the nominal direction of propagation z can be attributed to the (periodic) changes in the permittivities along this direction. The physical process that allows a layered metamaterial to mimic an anisotropic material is that surface plasmons are supported at an interface where the permittivity changes sign. When the metal permittivity is negative, the sign change occurs at every interface; the wave is transmitted via coupled surface plasmons [13]. MD stacks have potential applications, such as super-resolution with sub-wavelength focusing, negative refraction, harmonic generation, photonic bandgap structures and filters, sensing etc. [1,2,14–17].

Typically, EM propagation through MD stacks can be analyzed using the transfer matrix method (TMM) which is formulated for plane wave incidence, applied to the multilayered structure, or by using numerical methods, such as a finite element method, e.g. COMSOL. However, using effective medium theory and considering the MD stack to be an inhomogeneous bulk material, it is possible to use simpler and faster methods, such as the Berreman matrix method (BMM) to approximately

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https://doi.org/10.1016/j.optcom.2018.04.069
Received 13 March 2018; Received in revised form 25 April 2018; Accepted 27 April 2018

find the EM fields inside the structure and determine the transmission and reflection coefficients. Such an approach may be advantageous during the design of such MD stacks, where, say, the tuning behavior as a function of the wavelength or angle of incidence can be preliminarily assessed quickly using BMM. In this paper, illustrative examples of EM analysis of such structures using BMM are given and compared with results from TMM and COMSOL. Although it has been shown that the effective permittivity of such structures can be determined to within the Wiener bounds [18], it is instructive to determine how accurately effective medium theory accurately determines the transmittance and reflectance from MD stacks structures through direct simulations. It is concluded that BMM with effective medium results for an anisotropic material provides a fast and reliable estimate of the optical characteristics of the composite material. In the process, the concept of the angular plane wave matrix for propagation in anisotropic materials using BMM is introduced, and its application to beam propagation (including z-polarized beams) in such materials is discussed.

The organization of the paper is as follows. In Section 2, EM propagation using BMM in a bulk medium modeled as an effective medium is summarized, with emphasis on TM polarization and hyperbolic metamaterials. The concept of the transfer function matrix for propagation in such anisotropic metamaterials is introduced. In Section 3, TMM is summarized, along with a scheme to compare TMM with BMM for MD stacks. In Section 4, numerical results of BMM and TMM along with COMSOL are presented for transmittance and reflectance of MD stacks, especially those with dimensions which give rise to hyperbolic dispersion when modeled as an effective medium. It is shown that BMM provides a simple and fast way to get a general estimate of the spectral properties of such stacks for possible applications such as tunable filters, which can aid in the design of such structures for a given application. At the same time, through numerical simulations, it is shown why and how TMM results converge to that obtained from BMM using effective medium theory, as is expected from theoretical limits. Section 5 concludes the paper.

2. EM analysis using BMM for effective medium

As stated earlier, a metamaterial with hyperbolic dispersion can be built, for instance, as a multilayer structure consists of alternating layers of dielectric and metal (see Fig. 1(a)), and modeled as an anisotropic bulk medium BM (see Fig. 1(b)), based on the effective medium theory. Effective medium theory is a consequence of the homogenization technique [19,20]. This technique is based on the averaging of the EM field in the unit cell of metamaterial and can be applied to multilayered periodic systems. In addition, mean-field homogenization theories can also explain the effective parameters from the distribution of fundamental metamaterial enclosures, such as in Lorentz, Clausius–Mossotti, and Maxwell–Garnett approximations [18,19]. Rapidly-varying spatial scales and spatial periodicity are the two basic ingredients for homogenization approach of a metamaterial [20]. If the multilayered MD stack system, such as that shown in Fig. 1, is indeed periodic (i.e., the number of layers approaches infinity) and the metallo-dielectric patterning has a spatial scale which is much smaller than radiation wavelength, then one can treat the system as a bulk anisotropic medium with an effective dielectric permittivity tensor [1,13,21]

$$\left[\epsilon_{eff}\right] = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix}, \qquad \epsilon_{xx} = \epsilon_{yy} = \epsilon_0 \frac{\epsilon_1 d_1 + \epsilon_2 d_2}{d_1 + d_2},$$
$$\epsilon_{zz} = \epsilon_0 \frac{\left(\epsilon_1 \epsilon_2\right)\left(d_1 + d_2\right)}{\left(d_1 \epsilon_2 + d_2 \epsilon_1\right)}, \tag{1}$$

where d_1 and d_2 are the thickness of the dielectric and metal layers, respectively, ϵ_0 is the permittivity of free space and $\epsilon_1 = n_1^2$, $\epsilon_2 = n_2^2$ are relative permittivities of the dielectric and the metal, respectively. For future reference, it is useful to note that as long as the ratio d_1/d_2 is maintained a constant, the values of ϵ_{xx} and ϵ_{zz} remain unchanged. The dispersion relation for this anisotropic metamaterial is

$$\frac{k_x^2}{\epsilon_{zz}} + \frac{k_z^2}{\epsilon_{xx}} = \frac{k_0^2}{\epsilon_0}, \tag{2}$$

where k_x and k_z are the transverse and longitudinal components of the wave vector. If the condition $\epsilon_{zz} < 0$ and $\epsilon_{xx} > 0$, the dispersion relation is hyperbolic [1]. It is remarked that assuming real values for ϵ_1 and ϵ_2, hyperbolic dispersion can be obtained when the condition $\frac{\epsilon_1}{\epsilon_2} \geq -\frac{d_1}{d_2}$ is satisfied [1,22,23].

We consider a plane wave obliquely incident from an isotropic ambient medium (assumed to be free space) onto the anisotropic medium, finally exiting into free space once again. It is remarked that this technique can be readily extended to the case of arbitrary incident and transmitted media. The plane of incidence is the x–z plane, and we assume there is no variation in y direction and wave is propagating in the x–z direction. The bounds of the effective medium are $z = [0, L]$. The x- variation of all fields in all regions (a,b,c) is in the form $\exp(-jk_x x)$ where $k_x = k_0 sin\theta_i$, where k_0 is the propagation constant in free space, and θ_i is the angle of incidence from free space onto the medium, as shown in Fig. 1(b). This is because the momentum of the waves along the x-direction is unchanged since there is no interface normal to the x-direction. The incident magnetic field for TM polarization can be written as [22,23]

$$\boldsymbol{H_i} = \hat{\boldsymbol{a}}_{\boldsymbol{y}} H_a^+ e^{-jk_0\left(x sin\theta_i + z cos\theta_i\right)}, \tag{3}$$

where, H_a^+, k_0 and θ_i are the amplitude of the incident magnetic field, the free space wavenumber, and the angle of incidence, respectively. The reflected and transmitted magnetic fields, $\boldsymbol{H_r}$ and $\boldsymbol{H_t}$, respectively, can be represented in a similar way. The corresponding incident, reflected and transmitted electric fields, $\boldsymbol{E_i}$, $\boldsymbol{E_r}$, and $\boldsymbol{E_t}$ can be similarly written as

$$\boldsymbol{E_i} = (\hat{\boldsymbol{a}}_{\boldsymbol{x}} \cos\theta_i - \hat{\boldsymbol{a}}_{\boldsymbol{z}} \sin\theta_i) E_a^+ e^{-jk_0\left(x sin\theta_i + z cos\theta_i\right)}. \tag{4}$$

$$\boldsymbol{E_r} = (\hat{\boldsymbol{a}}_{\boldsymbol{x}} \cos\theta_i + \hat{\boldsymbol{a}}_{\boldsymbol{z}} \sin\theta_i) E_a^- e^{-jk_0\left(x sin\theta_r - z cos\theta_r\right)}, \tag{5}$$

$$\boldsymbol{E_t} = (\hat{\boldsymbol{a}}_{\boldsymbol{x}} \cos\theta_t - \hat{\boldsymbol{a}}_{\boldsymbol{z}} \sin\theta_t) E_c^+ e^{-jk_0\left(x sin\theta_t + (z-L) cos\theta_t\right)}. \tag{6}$$

$$M_B = j \begin{bmatrix} k_x \dfrac{\epsilon_{zx}}{\epsilon_{zz}} & \omega_0\left(-\mu_{yy} + \dfrac{\mu_{yz}\mu_{zy}}{\mu_{zz}} + \dfrac{{k_x}^2}{\omega_0^2 \epsilon_{zz}}\right) & k_x\left(\dfrac{\epsilon_{zy}}{\epsilon_{zz}} - \dfrac{\mu_{yz}}{\mu_{zz}}\right) & \omega_0\left(\mu_{yx} - \dfrac{\mu_{yz}\mu_{zx}}{\mu_{zz}}\right) \\ \omega_0\left(-\epsilon_{xx} + \dfrac{\epsilon_{xz}\epsilon_{zx}}{\epsilon_{zz}}\right) & k_x \dfrac{\epsilon_{xz}}{\epsilon_{zz}} & \omega_0\left(-\epsilon_{xy} + \dfrac{\epsilon_{xz}\epsilon_{zy}}{\epsilon_{zz}}\right) & 0 \\ 0 & \omega_0\left(\mu_{xy} - \dfrac{\mu_{xz}\mu_{zy}}{\mu_{33}}\right) & k_x \dfrac{\mu_{xz}}{\mu_{zz}} & \omega_0\left(-\mu_{xx} + \dfrac{\mu_{xz}\mu_{zx}}{\mu_{zz}}\right) \\ \omega_0\left(-\epsilon_{yx} + \dfrac{\epsilon_{yz}\epsilon_{zx}}{\epsilon_{zz}}\right) & k_x\left(\dfrac{\epsilon_{yz}}{\epsilon_{zz}} - \dfrac{\mu_{zy}}{\mu_{zz}}\right) & \omega_0\left(-\epsilon_{yy} + \dfrac{\epsilon_{yz}\epsilon_{zy}}{\epsilon_{zz}} + \dfrac{{k_x}^2}{\omega_0^2 \mu_{zz}}\right) & -k_x \dfrac{\mu_{zx}}{\mu_{zz}} \end{bmatrix} \tag{7}$$

Box I.