Optics Communications 426 (2018) 63–69

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Spontaneous emission of a moving atom in the presence of magnetodielectric material: A relativistic approach

Fariba Shafieiyan [a], Ehsan Amooghorban [a,b,c,*], Ali Mahdifar [a,b,d]

[a] Department of Physics, Faculty of Basic Sciences, Shahrekord University, P.O. Box 115, Shahrekord 88186-34141, Iran
[b] Photonic Research Group, Shahrekord University, Shahrekord 88186-34141, Iran
[c] Nanotechnology Research Center, Shahrekord University, Shahrekord 88186-34141, Iran
[d] Department of Physics, Faculty of Science, University of Isfahan, Hezar Jerib, Isfahan 81746-73441, Iran

ARTICLE INFO

Keywords:
Canonical quantization
Spontaneous emission
Multipolar Hamiltonian
Röntgen interaction

ABSTRACT

In this paper, based on a canonical quantization scheme, we study the effect of the relativistic motion of an excited atom on its decay rate in the presence of absorbing and dispersive media. For this purpose, we introduce an appropriate Lagrangian and describe the center-of-mass dynamical variables by the Dirac field. We obtain the Hamiltonian of the system in a multipolar form and calculate the motion equations of the system in the Schrödinger picture. We find that the decay rate and the quantum electrodynamics level shift of the moving atom can be expressed in terms of the imaginary part of the classical Green tensor and the center-of-mass velocity of the atom.

1. Introduction

One of the most fundamental phenomena in quantum optics is the spontaneous emission caused by the inevitable interaction of an excited atom with the vacuum-quantized electromagnetic field and/or the reaction of the atom to its own radiation field [1]. This process was first formulated theoretically by Dirac in 1927 and further by Weiskopff in 1930 [1]. In order to study the spontaneous emission, usually the atom is assumed to be in rest, which leads to difficulties due to Heisenberg's uncertainty relation [2]. In recent years there has been an increasing number of papers on the role of the center-of-mass motion in the process of spontaneous emission [3–5], Abraham–Minkowski-controversy [6–8], Aharonov–Bohm-type phase shifts [9–11] and many important effects and applications associated with atomic motion in atom optics, laser cooling, trapping, and isotope separation experiments [12].

By taking into account the so-called Röntgen term in the atom–field interaction, Wilkens evaluated the velocity dependence of the spontaneous decay rate of an atom which moves in free space with a constant velocity v to lowest order of v/c [3]. Later, Boussiakou et al. used a rigorous canonical formalism in which the center-of-mass dynamics of the atom is explicitly included and calculated the spontaneous decay of a moving excited atom in free space [4]. They showed that, irrespective of the orientation of the atomic dipole with respect to the direction of motion, the decay rate of the atom from the point of view of an observer in the laboratory frame is in agreement with special relativity. This result has been confirmed by an alternative but less general approach based on the basic principles of special relativity, physical processes associated with a moving electric dipole and the Doppler shift [5]. As a matter of fact, for more realistic cases atoms are not in free space, but move near the material media. Therefore, this is not a practical assumption for most real world applications.

It is well known that the presence of material media can change the structure of the fluctuating field of the vacuum. Consequently, the spontaneous emission rate can be modified if the atom moves with uniform nonrelativistic speed near materials of different composition and shape. In Refs. [13–15], authors considered a more realistic case and studied the spontaneous emission and the friction force experienced by an atom moving with uniform nonrelativistic velocity parallel to a dielectric surface. However, the question that naturally arises in this context is on the emission process occurring when an atom moves in absorbing magnetodielectric material relativistically. It is expected that the relativistic motion of the atom affects the radiative properties of the atom. The present paper is intended to respond this question. Our work extends previous works on the spontaneous decay of the moving excited atom in free space [4,5], to the relativistic motion in the presence of the material media.

As a first step in studying the relativistic dynamics of a moving atom in the presence of absorbing media one has to provide the

* Corresponding author at: Department of Physics, Faculty of Basic Sciences, Shahrekord University, P.O. Box 115, Shahrekord 88186-34141, Iran.
E-mail address: Ehsan.amooghorban@sci.sku.ac.ir (E. Amooghorban).

https://doi.org/10.1016/j.optcom.2018.05.037
Received 24 March 2018; Received in revised form 9 May 2018; Accepted 11 May 2018

quantization of the electromagnetic field. Generally speaking, there are two approaches to quantize the electromagnetic field in the presence of material media: phenomenological and canonical approach. In this paper, we follow the canonical methods as presented in [16–18]. More details concerning this rigorous canonical approach can be found in Refs. [16–23].

The paper is organized as follows. In Section 2, we present a canonical quantization of the electromagnetic field interacting with moving charge particles in the presence of an isotropic, inhomogeneous and absorbing magnetodielectric medium. We start from a convenient Lagrangian and obtain the canonical momenta and the Hamiltonian of the combined system. We apply this approach to the case of two non-relativistic particles of opposite charges which form an atomic system. On the Hamiltonian, we perform a unitary transformation and derive nonrelativistic multipolar Hamiltonian in the electric-dipole approximation. In Section 3, we generalize the formalism by introducing the Dirac field to describe the relativistic motion of the atom. In Section 4, we examine the time evolution of the atomic system by treating the atom's external and internal degrees of freedom on the same quantum footing in the Schrödinger picture. In this section, the decay rate and the quantum electrodynamics level shift of the two-level atom, that moving with relativistic velocity near dissipative media, are explicitly evaluated. Finally, the main results are summarized in Section 5.

2. Basic equations for non-relativistic dynamics

Let us consider a system composed of charged particles, the electromagnetic field, an absorbing and dispersive magnetodielectric medium and the interactions between them. Since, for short particle-medium separations the macroscopic description of the medium is not justified, we assume that the charged particles are placed in the free space and well separated from the medium. The Lagrangian of the whole system is written as follows [16–19,21–23]

$$L = L_q + L_{em} + L_m + L_{int}, \tag{1}$$

where

$$L_q = \frac{1}{2}\sum_\alpha m_\alpha \dot{\mathbf{r}}_\alpha^2(t), \tag{2}$$

is the Lagrangian for the charged particles with masses m_α, charges e_α and the position vector $\mathbf{r}_\alpha$, and the Lagrangian of the electromagnetic field, L_{em}, is given by $L_{em} = \frac{1}{2}\int d^3\mathbf{r}\left(\varepsilon_0\,\mathcal{E}^2(\mathbf{r},t) - \frac{\mathcal{B}^2(\mathbf{r},t)}{\mu_0}\right)$. Here, the electric and magnetic fields can be defined in terms of the vector potential $\mathbf{A}$ and the scalar potential φ as $\mathcal{E} = -\nabla\varphi - \frac{\partial \mathbf{A}}{\partial t}$ and $\mathcal{B} = \nabla\times\mathbf{A}$, respectively. In the Coulomb gauge, respectively, $-\nabla\varphi$ and $-\frac{\partial\mathbf{A}}{\partial t}$ are related to the longitudinal part $\mathcal{E}^{\parallel}$ and the transverse part $\mathcal{E}^{\perp}$ of the total electric field $\mathcal{E}$.

The third term in Eq. (1), L_m, denotes the magnetodielectric medium part as

$$L_m = \frac{1}{2}\int d^3\mathbf{r}\int_0^\infty d\omega\left[\dot{\mathbf{X}}_\omega^2(\mathbf{r},t) + \dot{\mathbf{Y}}_\omega^2(\mathbf{r},t) - \omega^2\left(\mathbf{X}_\omega^2(\mathbf{r},t) + \mathbf{Y}_\omega^2(\mathbf{r},t)\right)\right]. \tag{3}$$

Here, the medium is modeled by two independent sets of harmonic oscillators characterized by means of two medium fields $\mathbf{X}_\omega$ and $\mathbf{Y}_\omega$. This scheme is based on Hopfield's microscopic model [24], which provide the dissipation of the energy as well as the polarizability and the magnetizability characters of the medium.

Finally, the interaction part of the Lagrangian (1) is given by

$$\begin{aligned} L_{int} &= \sum_\alpha\left[e_\alpha\dot{\mathbf{r}}_\alpha\cdot\mathbf{A}(\mathbf{r}_\alpha,t) - e_\alpha\varphi(\mathbf{r}_\alpha,t)\right] \\ &+ \int d^3\mathbf{r}\,(\mathbf{P}(\mathbf{r},t)\cdot\mathcal{E}(\mathbf{r},t) + \mathbf{M}(\mathbf{r},t)\cdot\mathcal{B}(\mathbf{r},t)), \end{aligned} \tag{4}$$

where the terms in the first line describe the interaction of charged particles with the electromagnetic field, and those in the second line represent the interaction between the electromagnetic field and the material fields with the polarization vector $\mathbf{P}$ and the magnetization vector $\mathbf{M}$, respectively. Polarization and magnetization vectors can be, respectively, expressed in terms of the electric coupling function, g_e, and the magnetic coupling function, g_m, as follows

$$\mathbf{P}(\mathbf{r},t) = \int_0^\infty d\omega g_e(\mathbf{r},\omega)\mathbf{X}_\omega(\mathbf{r},t), \tag{5}$$

$$\mathbf{M}(\mathbf{r},t) = \int_0^\infty d\omega g_m(\mathbf{r},\omega)\mathbf{Y}_\omega(\mathbf{r},t). \tag{6}$$

We will see later that the dielectric permeability and the magnetic permittivity of the medium can be naturally expressed in terms of these coupling functions. To simplify the calculations, without loss of generality, we assume that the medium is isotropic. Therefore, the coupling functions g_e and g_m are both scalars, but take on tensor forms when the medium is anisotropic [18].

From the Lagrangian density (1), the canonical conjugate momenta associated to each dynamical variables can be obtained as

$$\mathbf{p}_\alpha(t) = \frac{\partial L}{\partial\dot{\mathbf{r}}_\alpha} = m_\alpha\dot{\mathbf{r}}_\alpha + e_\alpha\mathbf{A}(\mathbf{r}_\alpha,t), \tag{7}$$

$$-\varepsilon_0\mathcal{E}^{\perp}(\mathbf{r},t) = \frac{\delta L}{\delta\dot{\mathbf{A}}(\mathbf{r},t)} = \varepsilon_0\dot{\mathbf{A}}(\mathbf{r},t), \tag{8}$$

$$\mathbf{Q}_\omega(\mathbf{r},t) = \frac{\delta L}{\delta\dot{\mathbf{X}}_\omega(\mathbf{r},t)} = \dot{\mathbf{X}}_\omega(\mathbf{r},t) + g_e(\mathbf{r},\omega)\mathbf{A}(\mathbf{r},t), \tag{9}$$

$$\mathbf{\Pi}_\omega(\mathbf{r},t) = \frac{\delta L}{\delta\dot{\mathbf{Y}}_\omega(\mathbf{r},t)} = \dot{\mathbf{Y}}_\omega(\mathbf{r},t). \tag{10}$$

To describe the system quantum mechanically, we follow the standard canonical quantization procedure and impose between the variables and their canonical conjugates, which are now operators on the Hilbert space, the following commutation relations

$$[\hat{\mathbf{r}}_\alpha(t)\ \ \hat{\mathbf{p}}_\beta(t)] = i\hbar\delta_{\alpha\beta}, \tag{11}$$

$$[\hat{\mathbf{A}}(\mathbf{r},t)\ \ -\varepsilon_0\,\hat{\mathcal{E}}^{\perp}(\mathbf{r}',t)] = i\hbar\delta^{\perp}(\mathbf{r}-\mathbf{r}'), \tag{12}$$

$$[\hat{\mathbf{X}}_\omega(\mathbf{r},t)\ \ \hat{\mathbf{Q}}_{\omega'}(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'), \tag{13}$$

$$[\hat{\mathbf{Y}}_\omega(\mathbf{r},t)\ \ \hat{\mathbf{\Pi}}_{\omega'}(\mathbf{r}',t)] = i\hbar\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'), \tag{14}$$

whereas all the other commutators of the canonical variables vanish. With the help of the above canonical momenta, we can now derive the Hamiltonian of the system. After some algebra it reads

$$\begin{aligned} \hat{H} &= \sum_\alpha\frac{[\hat{\mathbf{p}}_\alpha - e_\alpha\hat{\mathbf{A}}(\hat{\mathbf{r}}_\alpha,t)]^2}{2m_\alpha} + \frac{1}{2}\int d^3\mathbf{r}[\varepsilon_0\,\hat{\mathcal{E}}^{\perp 2}(\mathbf{r},t) + \frac{\hat{\mathcal{B}}^2(\mathbf{r},t)}{\mu_0}] \\ &+ \frac{1}{2}\int d^3\mathbf{r}\int_0^\infty d\omega\,[\hat{\mathbf{Q}}_\omega^2(\mathbf{r},t) + \omega^2\hat{\mathbf{X}}_\omega^2(\mathbf{r},t)] \\ &+ \frac{1}{2}\int d^3\mathbf{r}\int_0^\infty d\omega\,[\hat{\mathbf{\Pi}}_\omega^2(\mathbf{r},t) + \omega^2\hat{\mathbf{Y}}_\omega^2(\mathbf{r},t)] \\ &- \int d^3\mathbf{r}[\hat{\mathbf{M}}(\mathbf{r},t)\cdot\hat{\mathcal{B}}(\mathbf{r},t) + \hat{\mathbf{P}}(\mathbf{r},t)\cdot\hat{\mathbf{A}}(\mathbf{r},t)] \\ &- \frac{1}{2}\int d^3\mathbf{r}\int_0^\infty d\omega\,g_e(\mathbf{r},\omega)\hat{\mathbf{A}}^2(\mathbf{r},t) + w_{coul}, \end{aligned} \tag{15}$$

where the Coulomb energy, w_{coul}, is due to the interactions between the charged particles, the charged particles and the polarization charges, and the interactions between the polarization charges, and is defined as follows [18]

$$\begin{aligned} w_{coul} &= \frac{1}{2}\int d^3\mathbf{r}\,\hat{\rho}_A(\mathbf{r})\hat{\varphi}_A(\mathbf{r}) + \int d^3\mathbf{r}\,\hat{\rho}_A(\mathbf{r})\hat{\varphi}_P(\mathbf{r}) \\ &+ \frac{1}{2}\int d^3\mathbf{r}\,\hat{\rho}_P(\mathbf{r})\hat{\varphi}_P(\mathbf{r}). \end{aligned} \tag{16}$$

Here, $\hat{\rho}_A(\mathbf{r}) = \sum_\alpha e_\alpha\delta(\mathbf{r}-\hat{\mathbf{r}}_\alpha)$, $\hat{\rho}_P(\mathbf{r}) = -\nabla\cdot\hat{\mathbf{P}}(\mathbf{r})$, are, respectively, the charge density, the polarization charge density, and the scalar potential which are attributed to the external and the polarization charges,