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Analytical study of the self-reconstruction of a partially coherent Gaussian Schell-model beam

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ABSTRACT

An analytical expression for partially coherent Gaussian Schell-model (GSM) beams partially blocked by an opaque obstacle is derived. Based on the formula, the self-reconstruction is studied in detail. It is found that the reconstruction process, characterized by the reconfiguration distance that is defined in this paper, is long when the beam size is large or the obstacle size is large or the coherence length is large. Moreover, we find that there is one condition for the existence of the self-reconstruction effect of a partially coherent GSM beam. Our findings are useful in optical tweezers and optical trapping.

1. Introduction

It has been known that some types of light beams possess the ability of self-reconstructing in the presence of obstacles, such as Bessel beams [1,2], Airy beams [3,4], Mathieu beams [5] and caustic beams [6]. Those beams are called diffraction-free beams. However, it was gradually shown that some diffracting coherent beams, including optical ring lattices [7], vector Laguerre–Gaussian beams [8], tightly focused [9] and radially polarized [10] Bessel–Gauss beams and Pearcey beams [11], can also be self-healing. The self-healing property has attracted a lot of attentions because of its useful applications. Garcés-Chávez et al. showed that the reconstructive property may be utilized within optical tweezers to trap particles in multiple, spatially separated sample cells with a single beam [12]. Fahrbach et al. have applied the self reconstructive property to the microscope [13] and they found that a Bessel beam is unexpectedly robust against deflection from objects. McLaren et al. studied the quantum entanglement propagation in the presence of obstructions and confirmed that, for the entanglement of orbital angular momentum, measurement in the Bessel basis is more robust to losses than measuring in the usually employed Laguerre–Gaussian basis [14]. The application of nondiffracting beams to wireless optical communications has been investigated [15]. All the researches mentioned above are about the fully coherent beams. In a recent paper, partially coherent beams have been experimentally observed that have the self-reconstruction ability [16,17].

Partially coherent beams have become increasingly popular in light propagation and modulation. Spatial coherence has proven to be an effective degree of freedom to control the light beam propagation. Its importance is the same as the amplitude, phase and polarization of light. Partially coherent beams with special spatial coherence have novel properties. For example, they can form a ring shaped spot [18,19] or flat-top intensity profile [20] in the far zone. They can form far-field optical lattice patterns [21,22]. They can be self-splitting in propagation [23,24]. Partially coherent beams have important applications. It has been shown that the coherence of an incident beam affects the beam propagation in turbulent atmosphere and have demonstrated that low coherent sources were less influenced by atmospheric turbulence [25,26]. Raghunathan et al. have shown that manipulation of the coherence of the incident beam can be used to shape the intensity distribution in the focal region which is useful in optical tweezers and in optical trapping [27]. Other applications of partially coherent beams have been reported, including ghost imaging [28], coherence holography [29], and diffractive imaging [30]. The reconstruction of scalar and vector partially coherent beams were reported in [31] and [17], respectively. They demonstrated that partially coherent beams are unexpectedly robust against scattering by objects [31]. A vector partially coherent beam, scattered by an arbitrary opaque obstacle, can recover its intensity profile and the state of polarization [17]. The authors in Refs. [17] and [31] adopted numerical calculation and experimental demonstration to study the self reconstruction of partially coherent beams, but it is very important to obtain an analytical expression. This is because an analytic expression can be used to easily and clearly analyze the self-reconstruction process and the effects of related factors. Very recently, two papers reported the self-reconstruction of partially coherent beams [32,33]. One paper focused on the self-construction of the degree of coherence [32]. The other paper paid attention to

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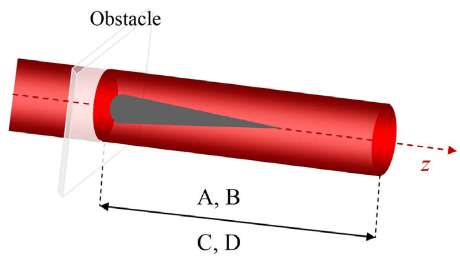

Fig. 1. Illustrating the notation. The red cylinder denotes the laser beam, the black cone denotes the shadow behind the obstacle.

charactering the self-reconstruction ability [33]. Almost all the papers on the self-reconstruction of partially coherent beams mentioned above do not analyze the physical process in detail. In the present paper, we will analytically study the self-reconstruction of partially coherent GSM beams and give the condition of self-reconstruction existence for those beams. Moreover, according to the analytical formula, we will analyze the physical process of self-reconstruction and give the reconfiguration distance.

2. Analytical expression for a partially coherent beam partially blocked by an opaque obstacle

Let us consider a partially coherent beam, partially blocked by an obstacle, propagating close to the z direction into a ABCD optical system (see Fig. 1). The source plane is defined in plane $z = 0$. The statistical property of a partially coherent beam in the source plane can be characterized by the cross-spectral density (CSD) [34]

$$W(\mathbf{r}_1,\mathbf{r}_2) = \langle E^*(\mathbf{r}_1)E(\mathbf{r}_2)\rangle, \tag{1}$$

where $E(\mathbf{r})$ is the electric field of light, $\mathbf{r} = (\mathbf{x},\mathbf{y})$ is the transverse position vector at the source plane, the angle brackets denote the ensemble average, and the star is the complex conjugate. One can obtain the average intensity at one point $\mathbf{r}$ based on the CSD [35]

$$\langle I(\mathbf{r})\rangle = W(\mathbf{r},\mathbf{r}). \tag{2}$$

To analytically study the self-reconstruction properties of the partially coherent beams, we assume, for convenience, that a disk-shaped obstacle being positioned at $z = 0$ partially blocks the initial beam and it has Gaussian absorption efficiency. Therefore, the transmittance function can be expressed as

$$T(\mathbf{r}) = 1 - \exp\left(-\frac{\mathbf{r}^2}{\omega_d^2}\right), \tag{3}$$

where ω_d denotes the size of the obstacle. With the help of the generalized Huygens–Fresnel integral, the average intensity at the receiver plane can be expressed as

$$\langle I(\rho,z)\rangle = \left(\frac{1}{\lambda B}\right)^2 \iiiint T^*(\mathbf{r}_1)T(\mathbf{r}_2)W(\mathbf{r}_1,\mathbf{r}_2,0) \times \exp\left[-\frac{ik}{2B}[A(r_1^2-r_2^2) - 2(\mathbf{r}_1-\mathbf{r}_2)\cdot\rho]\right] d^2\mathbf{r}_1 d^2\mathbf{r}_2, \tag{4}$$

where $\rho = (u,v)$ is the transverse position vector at the receiver plane, $k = 2\pi/\lambda$ is the wavenumber with wavelength λ, A, B, C and D are the elements of the transform matrix of the optical system. We specifically consider the input beam to be of a Gaussian Schell-model type such that [36]

$$W(\mathbf{r}_1,\mathbf{r}_2,0) = \exp\left(-\frac{r_1^2+r_2^2}{\omega_0^2}\right)\exp\left(-\frac{(\mathbf{r}_1-\mathbf{r}_2)^2}{2\delta_0^2}\right), \tag{5}$$

where ω_0 is the beam waist size and δ_0 denotes the transverse coherence length at the source plane. On substituting Eqs. (3) and (5) into Eq. (4) and using a new coordinates $\mathbf{r}_s = (\mathbf{r}_1+\mathbf{r}_2)/2$, $\mathbf{r}_d = \mathbf{r}_1 - \mathbf{r}_2$, we obtain

$$\langle I(\rho,z)\rangle = \left(\frac{1}{\lambda B}\right)^2 \iint \tilde{T}(\mathbf{r}_d) \times \exp\left(-\frac{1}{2\omega_0^2}r_d^2 - \frac{1}{2\delta_0^2}r_d^2\right)\exp\left(\frac{ik}{B}\mathbf{r}_d\cdot\rho\right) d^2\mathbf{r}_d, \tag{6}$$

with

$$\tilde{T}(\mathbf{r}_d) = \iint \Bigg[1 - \exp\left(-\frac{r_s^2}{\omega_d^2} - \frac{\mathbf{r}_s\cdot\mathbf{r}_d}{\omega_d^2} - \frac{r_d^2}{4\omega_d^2}\right) - \exp\left(-\frac{r_s^2}{\omega_d^2} + \frac{\mathbf{r}_s\cdot\mathbf{r}_d}{\omega_d^2} - \frac{r_d^2}{4\omega_d^2}\right) + \exp\left(-\frac{2}{\omega_d^2}r_s^2 - \frac{r_d^2}{2\omega_d^2}\right)\Bigg] \exp\left(-\frac{2}{\omega_0^2}r_s^2\right)\exp\left(-\frac{ikA}{B}\mathbf{r}_d\cdot\mathbf{r}_s\right) d^2\mathbf{r}_s. \tag{7}$$

After the integral in Eq. (7), we have

$$\tilde{T}(\mathbf{r}_d) = \frac{\pi\omega_0^2}{2}\exp\left(-\frac{k^2A^2\omega_0^2r_d^2}{8B^2}\right) - \frac{2\pi\omega_d^2\omega_0^2}{2\omega_d^2+\omega_0^2} \times \exp\left(-\frac{(2B^2+k^2A^2\omega_0^2\omega_d^2)r_d^2}{4B^2(2\omega_d^2+\omega_0^2)}\right)\cos\left[\frac{kA\omega_0^2r_d^2}{2B(2\omega_d^2+\omega_0^2)}\right] + \frac{\pi\omega_0^2\omega_d^2}{2(\omega_d^2+\omega_0^2)}\exp\left(-\frac{[k^2A^2\omega_d^2\omega_0^2\omega_d^2+4B^2(\omega_d^2+\omega_0^2)]r_d^2}{8B^2\omega_d^2(\omega_d^2+\omega_0^2)}\right). \tag{8}$$

On substituting from Eq. (8) into Eq. (6) and carrying out the integration, we obtain

$$\langle I(\rho,z)\rangle = \frac{1}{\Delta^2(z)}\exp\left(-\frac{2\rho^2}{\omega_0^2\Delta^2(z)}\right) - 2Re\left[\frac{1}{\Delta_d'^2}\exp\left(-\frac{\rho^2}{\omega_d'^2\Delta_d'^2}\right)\right] + \frac{1}{\Delta_d^2(z)}\exp\left(-\frac{2\rho^2}{\omega_0'^2\Delta_d^2(z)}\right), \tag{9}$$

with

$$\frac{1}{\Omega^2} = \frac{1}{\omega_0^2} + \frac{1}{\delta_0^2};\ \frac{1}{\Omega_d^2} = \frac{1}{\omega_0'^2} + \frac{1}{\delta_0^2}, \tag{10}$$

$$\frac{1}{\omega_0'^2} = \frac{1}{\omega_0^2} + \frac{1}{\omega_d^2};\ \frac{1}{\omega_d'^2} = \frac{2}{\omega_0^2} + \frac{1}{\omega_d^2}, \tag{11}$$

$$\Delta^2(z) = A^2 + \frac{4B^2}{k^2\omega_0^2\Omega^2};\ \Delta_d^2(z) = A^2 + \frac{4B^2}{k^2\omega_d'^2\Omega_d^2}, \tag{12}$$

$$\Delta_d'^2(z) = \left[\frac{1}{2\omega_d'^2}\frac{1}{\Omega^2} + \frac{1}{2\omega_0^2\omega_d^2} + \frac{k^2A^2}{4B^2} - \frac{ikA}{2B\omega_d^2}\right]\frac{4B^2}{k^2}. \tag{13}$$

With the help of Eq. (9), we can study the self-reconstruction properties of a partially coherent GSM beam, partially blocked by an opaque obstacle, as it propagates in free space. Hence the elements of the transform matrix is $A = D = 1, B = z, C = 0$.

In the following numerical examples, we set $\lambda = 632.8$ nm. Fig. 2 shows the contour plot of the intensity distribution on the x–z plane. It is clearly seen the self-reconstruction process that the defect of the partially coherent GSM beam is gradually reconstructed during the propagation with a short distance (see b-2 and c-2), but there is no obvious evidence of reconstruction for the fully coherent GSM beam at the same propagating distance (see a-2). One can also find from this panel that the propagating distance between 0 and 500 mm looks like the self-reconstruction process, but after this distance, one central peak gradually emerges and almost divides the beam into three parts in the transverse plane. Therefore, the whole process does not represent a self-reconstruction. This is because the fully coherent GSM beam does not have the self-reconstruction ability. It is well known that for