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Stability of discrete vortex solitons in linear scaled-space square lattices

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ABSTRACT

We report the existence and stability of 1-charge discrete vortex solitons in linear scaled-space square lattices with photorefractive self-focusing nonlinearity. By scaling the square lattice along the edge and diagonal, we obtained rectangular and diamond lattices. In both settings, it is shown that the vortices can be stable in a moderate power region, and the vortices with high power are unstable, suffering from oscillatory instabilities. The structure of the rectangular potential strongly affects the profile of the vortex solitons, leading to distinct intensity asymmetry, phase dislocation, and exponential instability in the low power region. Correspondingly, the vortex profile could be well maintained in diamond lattices. Fascinating, even the low power unstable vortices suffered from both oscillatory and exponential instability in diamond lattices, they have a much weaker instability than counterparts in rectangular lattices.

1. Introduction

Vortex solitons are special nonlinear localised states that appear in various branches of physics, such as optics, and Bose–Einstein condensates [1]. An optical vortex is one type of optical singularity which possesses a helical wave-front structure and a phase singularity in the centre [2]. Due to the azimuthal modulational instability, vortexes with orbital angular momentum are unstable in a self-focusing homogeneous nonlinear medium and will break into several fundamental solitons that fly off tangential to the vortex ring [3]. Stable optical vortex solitons have been predicted in media with competing nonlinearities [4,5], nonlocal nonlinearities [6], alternating self-focusing and self-defocusing layers in Kerr media [7], periodic and ring-like potentials [8], parity-time-symmetric potentials [9], etc. Photonic lattices (e.g., a periodic structure of refractive index modulation) result in unique photonic band-gap structures similar to Bloch bands in solid-state physics that have served as an important tool for wave control [10]. Vortex solitons have been found in self-focusing media with photonic lattices called discrete vortex solitons [11], in which the main energy of soliton focus on the lattice sites like several fundamental solitons with a vortex phase structure and the propagation constant located in the semi-infinite Bloch gap. Stable 1-charge discrete vortex solitons have been predicted to exist theoretically in Kerr media with square photonic lattices, using either discrete models or continuous models. Discrete vortex solitons were first predicted to exist in discrete models [11,12], where the NLSE(nonlinear Schrödinger equation) has a discrete form called DNLSE, and it was found that the soliton solution is stable when the inter-site coupling is smaller than a critical value [11,13].

In continuous models, the discrete vortex solitons can be classified by different propagation constants, and a stable soliton was found in a region of propagation constants depending on the lattice profile. The discrete vortex solitons and gap vortex solitons in photonic lattices have been studied systematically for both Kerr and saturable nonlinearities [14]. Two independent research groups simultaneously observed discrete vortex solitons in photorefractive self-focusing media with optically induced square lattices [15,16]. It is interesting to investigate the discrete vortex solitons in non-square photonic lattices such as hexagonal lattices and the interface of different lattices. In hexagonal lattices, 1-charge vortex solitons with six main sites have been found to be always unstable, but double-charge vortex solitons can be stable within a certain range [17,18]. Vortex solitons located in the interface of photonic lattices have been found with asymmetric energy and the phase distribution depends on the profiles of the lattice [19].

In this paper, we exploit the existence and stability of discrete vortex solitons in linear scaled-space square lattices within the framework of a continuous nonlinear model of optically induced lattices generated in photorefractive nonlinear media. We scaled the lattice along two directions, the edge direction and the diagonal direction. We reveal that the stable range of the soliton shrinks quickly with the increase of asymmetry introduced by scaling in the edge direction, and two critical points appear in the stretched and compressed case. In contrast to edge

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scaling, the soliton range will be narrow only when the lattice is scaled down in the diagonal direction. We also investigated how the soliton solution responds to lattice scaling and the instability modes in different unstable ranges.

2. Modelling of vortex solitons

According to a paraxial approximation, the propagation of a probe beam in a biased photorefractive crystal with optically induced photonic lattices is governed by the normalised (1+2) dimensional nonlinear equation with saturable nonlinearity:

$$i\frac{\partial U}{\partial Z}+\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}-\frac{E}{1+V+|U|^2}U=0 \quad (1)$$

where $U(X,Y,Z)$ is normalised by the slowly varying complex envelope of the probe beam. The transverse coordinates X, Y and longitudinal coordinate Z are dimensionless parameters, normalised by the period of the lattice and the diffraction length of the probe beam, respectively. The parameters E proportional to the bias voltage can be positive or negative. Here, we chose $E > 0$ corresponding to self-focusing nonlinearity. $V(X,Y)$ is a periodic lattice potential in the transverse coordinate. In experiment, the photonic lattices can optically induced by printing an ordinarily polarised non-diffractive pattern onto a biased photorefractive strontium barium niobate crystal (SBN), where the probe beam should be extraordinarily polarised. A similar equation was used to describe the saturation of the electronic Kerr effect for increasing intensities [20].

We study the existence and stability of discrete vortex solitons in linear scaled-space square lattices. For example, square lattices that are linear scaled along the edge can be expressed as:

$$V(X,Y,\alpha)=I\sin^2\frac{X}{\alpha}\sin^2 Y. \quad (2)$$

Here I is the peak intensity and α is the space scaling parameter along the edge.

For vortex solutions, we write the complex envelope in the stationary form as $U(X,Y,Z)=u(X,Y)e^{i\mu Z}$, where μ is the propagation constant, and $u(X,Y)$ is a complex-value function. We obtain a new nonlinear equation for $u(X,Y)$ by substituting the stationary form into Eq. (1):

$$\left(\frac{\partial^2}{\partial X^2}+\frac{\partial^2}{\partial Y^2}+\mu-\frac{E}{1+V+|u|^2}\right)u=0. \quad (3)$$

The solutions can be found by several numerical relaxation schemes with a guessed solution. We used the modified squared-operator method (MSOM) [21]. The initial guess is a vortex beam with a 2π phase accumulation around the phase singularity local to the centre of the square lattice unit, and the propagation constant μ local to the semi-infinite Bloch gap. By changing the scaling factor α and the scaling directions, we determine two classes of square-like discrete vortex solitons.

3. Rectangular vortex solitons

First, we present the properties of 1-charge off-site vortices in self-focusing photorefractive media with rectangular 2D periodic lattices. The modulation form $V = I\sin^2(X/\alpha)\sin^2 Y$ could be seen as scaled square lattices in the coordinate X (along the square edge), where α is the scaling parameter. We consider both the stretched and compressed square lattices corresponding to $\alpha > 1$ and $\alpha < 1$, respectively. We remind of the sites of the rectangular lattices that we obtained are an ellipse. To obtain a large stable region, we set $I = 2$ and $E = 7.5$ according to reference [22]. We illustrate the rectangular lattice potential with $\alpha = 1.3$ in Fig. 1(a). In square lattices, the off-site vortices have four exactly symmetrical major lobes located on the lattice sites and the phase singularity is at the centre of the four lobes (see Fig. 1(b1) and (b2), whereas the scaling of the lattice breaks the symmetry of the vortices in the diagonal direction. The vortex profiles in the rectangular lattices are not just scaled from those in the square lattices. Two examples corresponding to $\mu = -4.5$, $\alpha = 1.075$ and 1.15 are shown in Fig. 1(c1) and (d1). To show the asymmetry of the distinctly, we illustrate the corresponding slices (along the dashed line in Fig. 1(c1) and (d1)) of the vortex profiles along the rectangle length and width in Fig. 1(c2) and (d2). One can see that the profiles of the vortex intensities are no longer symmetrical for the diagonal, even though the four lobes have the same peak intensity. The lattice scaling turn the field distribution between the two neighbouring lobes and the trail lattice sites. There is more energy appearing between two neighbouring lobes in the lengthwise slice than that in the widthwise slice. The trails of the vortex behave different in the two slices. The trails of the vortices become lower in the lengthwise slice. Notably, the trails in the widthwise slice are larger even compared to those in the original square lattices.

The phase structure is the inherent feature in vortex solitons. In normal square lattices, the phase difference is $\pi/2$ between every two neighbouring lobes (see Fig. 1(b2)). In a rectangular lattice, the phase difference between the lobes in the lengthwise direction is smaller than $\pi/2$ (Fig. 1(c3)). At a scaling factor of $\alpha = 1.15$, the phase structure of the vortex with $\mu = -4.5$ is more likely due to the dipole soliton (Fig. 1(d3)). The four lobes can be divided into two parts by the phase difference.

The asymmetry clearly appears when the propagation constant of the vortices is near the first Bloch band. On this occasion, the vortices have lower intensity and possess more trails in the adjacent sites. The profiles of the vortex soliton intensity distributions with $\alpha = 1.1$, $\mu = -4.5$ and -3.5 are illustrated in Fig. 2(a1) and (b1). With a large propagation constant, the trails are compressed like that in square lattices and the profiles in both slices views tend to be equal (Fig. 2(b2)). With the scaling factor settled ($\alpha = 1.1$), the phase structure of the vortex with $\mu = -3.5$ is similar to the vortex in square lattices than the vortex with $\mu = -4.5$ (see Fig. 2 (a3) and (b3)).

To determine the stability of the soliton, we search the linear-stability spectrum of the solitary solutions in Eq. (1). If $U(X,Y,Z) = u(X,Y)e^{i\mu Z}$ is a solitary solution for Eq. (1), a perturbed solution can be expressed:

$$U_p(X,Y,Z)=\left\{u(X,Y)+[v(X,Y)-w(X,Y)]e^{\lambda Z} + [v(X,Y)+w(X,Y)]^* e^{\lambda^* Z}\right\}e^{i\mu Z} \quad (4)$$

where $v(X,Y)$ and $w(X,Y)$ are normal-mode perturbations and λ is the complex-valued instability growth rate. Substituting the perturbed solution into Eq. (1) and linearising, we obtain the following eigenvalue problem:

$$i\begin{pmatrix} G_0 & \nabla^2+G_1 \\ \nabla^2+G_2 & -G_0\end{pmatrix}\begin{pmatrix} v \\ w\end{pmatrix}=\lambda\begin{pmatrix} v \\ w\end{pmatrix} \quad (5)$$

where

$$G_0=\frac{1}{2}\left(u^2-u^{*2}\right)\frac{E}{\left(1+V+|u|^2\right)^2}$$

$$G_1=-\mu-\frac{E}{1+V+|u|^2}+\left[|u|^2-\frac{1}{2}\left(u^2-u^{*2}\right)\right]\frac{E}{\left(1+V+|u|^2\right)^2}$$

$$G_2=-\mu-\frac{E}{1+V+|u|^2}+\left[|u|^2+\frac{1}{2}\left(u^2-u^{*2}\right)\right]\frac{E}{\left(1+V+|u|^2\right)^2}.$$

According to Fourier collocation method [23], we obtain the stability spectrum of a solitary solution. A solution is stable only when Real(λ) = 0 for all of the eigenvalues λ in Eq. (5). We show the power versus propagation constant of the vortex solitons in Fig. 3(a). The power of the vortex solitons is defined as $P=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|u|^2dxdy$. The power is a monotone increasing function of the propagation constant. The profile of P–μ curve is unchanged with the scaling parameter α which is just have a small shift. With μ fixed, the vortices have lower power with a larger scaling factor in rectangular lattices, although the lattice sites have a larger area. We show the maximum growth-rate diagram for $\alpha = 1$ and $\alpha = 1.1$ in Fig. 3(b). The vortices in square lattices are linearly stable