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# Generation of high frequency trains of chirped soliton-like pulses in inhomogeneous and cascaded active fiber configurations

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ABSTRACT

We present a theoretical formalism of description of soliton-like pulse generation in longitudinally inhomogeneous active optical fibers. Dynamics of the wave packet during the nonlinear stage of the modulation instability is described for fibers with distributed gain and group velocity dispersion profiles. Generation of pico- and sub-picosecond pulse trains operating in THz repetition range from quasi-continuous pump wave is simulated for fibers of different inhomogeneity and for cascade fiber configurations comprising homogeneous and inhomogeneous segments. Pulse train generation is shown to occur in resonant manner once the gain and GDV distributed fiber profiles are perfectly matched.

## 1. Introduction

Generation of THz ultra-short pulse trains at terahertz repetition rate and higher is among the current topical technological challenges. The induced modulation instability (MI) is an effective way to generate short optical pulses in optical fibers with a precise control of the pulse repetition rate [1–5]. In this process [3,4] a harmonic modulation of the input wave packet (WP) transforms into a train of ultrashort pulses separated by the modulation period. Commonly, dynamics of the generated pulses exhibit periodic behavior and the maximum pulse peak power achievable through the process could slightly exceed the initial pump power level. To form a pulse train of a maximal contrast the WP has to pass the fiber length that is about ∼1.5 times of the dispersion length [3]. When passing longer fibers, the pulse structure collapses and finally returns to the original WP state.

In order to generate a stable pulse train with peak powers much higher than the initial pump power level by the means of MI, the effective incoherent optical amplification should be somehow incorporated into the fiber configuration. With homogeneous nonlinear active fibers the possibilities of incoherent amplification of solitons are quite limited. As the soliton initial energy increases *e* fold, independently of the way it is amplified, significant distortions of the soliton's shape and spectrum occur due to generation of non-solitonic components. As a result, the nonlinear WP loses its soliton properties becoming structurally unstable. For a long time, such behavior of the amplified soliton was commonly accepted as the only possible evolution. However, in [6–8], an efficient amplification of the optical soliton-like pulse (SLP) without modification of its shape has been demonstrated in situation when the initial SLP phase is a parabolic function of time and the gain increment is a hyperbolic function of a distance. Interaction of such frequency-modulated (FM) pulses becomes entirely elastic due to self-matching of the SLP phase and medium gain.

One of the main obstacles impeding an experimental realization of such an "ideal" SLP amplification is the need to implement the required gain profile along the fiber length. It has been shown, however, in [8–12] that for the effective SLP amplification one can use the fibers not only with a hyperbolic, but, practically, any chosen gain profile. The only condition for that is the distribution of the group velocity dispersion (GVD) along the fiber matching the given gain profile. The fibers with W-like cross section profile of the refractive index and anomalous GVD gradually decreasing along the fiber length seem to be a good candidate for this application. Such fibers could be designed and manufactured with the given GVD distribution along the fiber length ensuring negligible third-order dispersion parameter. Tuning of the GVD profile in these fibers is typically achieved through a controllable change of the fiber cross-section area. The current fiber drawing technology allows to control GVD slope within a significant range of values variation of the fiber diameter over the whole fiber length very small (typically <3%).

In the present work, we demonstrate generation of high-frequency trains of ultrashort pulses with the peak powers significantly exceeding the power of the initial pump-wave. The technique employs the

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nonlinear stage of MI (modulation amplitude is comparable with pump wave power) in active inhomogeneous optical fibers and fiber cascades enabling resonant generation of picosecond and sub-picosecond pulses once the initial WP parameters (power and chirp) and GDV distribution profile in the fiber are perfectly matched. In particular, the performed numerical simulations based on the Schrödinger equations explore the effect of the GVD distribution profile and active fiber gain spectrum width on formation of pico- and sub-picosecond laser pulse trains with THz repetition rate directly from the modulated CW wave.

It is worth noting that the possibility of formation, stable propagation and temporal compression of solitonic trains in inhomogeneous optical fiber configurations has been considered in [13–19]. Commonly, for formation of high repetitive rate pulse trains through modulation instability in an optical fiber a superposition of two monochromatic waves of nearly equal amplitudes and of two different frequencies are used as an input wave [15]. In contract to the previous works, while still employing inhomogeneous GVD fibers for high-frequency train generation, in this paper, we demonstrate resonant generation of frequency-modulated pico- and sub-picosecond SLP directly from a quasi-continuous initial wave. In particular, we highlight formation of a linear pulse chirp (parabolic phase) during the first stage of the pulse train development that makes possible their further effective soliton-like amplification up to pulse peak powers exceeding the power level of the initial wave by orders of magnitude. An important advance of the proposed method is the possibility of pico- and sub-pico-second periodic pulse train generation from the initially modulated wave with a very small modulation depth ($m \ll 10^{-3}$) that is orders of magnitude lower that the initial modulation depth ($m \sim 0.5-1$) used in early experiments mentioned above [14,15]. Potentially, it opens the way to produce ultra-short pulses through resonant MI arising from the spontaneous noise always present in the initial continues pump wave ($m \to 0$).

## 2. Dynamics of SLP amplification in inhomogeneous active fibers

Let us consider the dynamics of the optical WP propagating in an amplifying inhomogeneous optical fiber [20,21]. The electric wave field is given as

$$\mathbf{E}(t,r,z) = \frac{\mathbf{e}}{2}U(r,z)\left\{A(t,z)\exp\left[i\left(\omega_0 t - \int_0^z \beta'(\xi)d\xi\right)\right] + K.C.\right\}, \quad (1)$$

where $\mathbf{e}$ is the polarization unit vector, $U(r,z)$ is the radial field distribution along the fiber, $\omega_0$ is the WP carrier frequency, $\beta'$ is the real part of the propagation constant. The nonlinear Schrödinger equation [1–3] describing the temporal envelope amplitude $A(t,z)$ with the coefficients depending on the longitudinal coordinate could be presented as:

$$\frac{\partial A}{\partial z} - i\frac{D_2(z)}{2}\frac{\partial^2 A}{\partial \tau^2} + iR(z)|A|^2 A = \gamma(z)A. \quad (2)$$

Here, $\tau$ is the time in the running coordinate frame, $D_2(z) = (\partial^2\beta'(z)/\partial\omega^2)_0$ is GVD, $R(z)$ and $R(z)$ are the coefficient of Kerr nonlinearity and effective gain increment, respectively:

$$\gamma(z) = g(z) - (\partial S_m/\partial z)/2S_m - K(z), \quad (3)$$

where $g(z)$ is the gain increment determined by the optical fiber material, $S_m$ is the effective fiber mode area [1,3,4] and $K(z)$ is the fiber loss coefficient. The energy of the pulse propagating along the fiber is determined by the material gain increment as:

$$W(z) = W_0 \exp\left(2\int_0^z \gamma(\xi)d\xi\right), \quad (4)$$

where $W_0$ is the input pulse energy.

For a fiber with the constant anomalous GVD and nonlinearity, and the material gain increment distributed along the fiber length as

$$\gamma(z) = \gamma_0/(1-2\gamma_0 z) \quad (5)$$

the pulse with a hyperbolic secant profile amplified while $2\gamma_0 z < 1$ is a solution described by Eq. 2 [7,8]:

$$A(\tau,z) = A_0\frac{\mathrm{sech}(\tau/\tau_s)}{1-2g_0 z}\exp\left(i\frac{\alpha_0\tau^2 - \Gamma z}{1-2g_0 z}\right). \quad (6)$$

Here, $\tau_s = \tau_0(1-2\gamma_0 z)$, $\Gamma = \gamma_0/2\alpha_0\tau_0^2$ and we assume that $D_2 R < 0$ and $2\Gamma = |D_2|/\tau_0^2 = R|A_0|^2$.

Nonlinear WPs described by Eq. (6) possess the properties of elastic interaction that are important for practical applications and referred to as "light" FM solitons [7,8].

The dispersion and non-linearity coefficients varying along the fiber length could be expressed in dimensionless variables: $d(z) = D_2(z)/D_0$ and $r(z) = R(z)/R_0$, where $D_0$ and $R_0$ are the values at the fiber input. With $\eta(z) = \int_0^z d(\xi)d\xi$ and $C(\tau,z) = \sqrt{r/d}\,A(\tau,z)$ Eq. (2) transforms into the form:

$$\frac{\partial C}{\partial \eta} - i\frac{D_0}{2}\frac{\partial^2 C}{\partial \tau^2} + iR_0|C|^2 C = \gamma_{ef}(\eta)C. \quad (7)$$

Thus, the problem of nonlinear pulse propagation along the optical fiber with the material parameters varying along the fiber length is reduced to the problem of pulse propagation along the fiber with constant dispersion $D_0$ and nonlinearity $R_0$ but with the variable effective gain $\gamma_{ef}(\eta)$:

$$\gamma_{ef}(\eta) = \frac{g(\eta) - K(\eta)}{d(\eta)} - \frac{1}{2}\frac{\partial}{\partial\eta}\ln\frac{\widetilde{S}_m(\eta)d(\eta)}{r(\eta)}, \quad (8)$$

where $\widetilde{S}_m = S_m(\eta)/S_m(0)$ is the normalized effective mode area.

Similar to Eqs. (2), (7) has a solution in the form of the amplified frequency-modulated (FM) soliton under conditions that $D_2(\eta)R(\eta) < 0$ and the effective gain increment (8) expressed as $\gamma_{ef}(\eta) = b_0/(1-2q\eta)$, where $b_0 = \gamma_{ef}(0)$. In this case, the solution of Eq. (7) is:

$$C(\tau,\eta) = C_0\frac{\mathrm{sech}(\tau/\tau_s)}{1-2b_0\eta}\exp\left(i\frac{\alpha_0\tau^2 - \Gamma_0\eta}{1-2b_0\eta}\right), \quad (9)$$

where $\tau_s = \tau_0(1-2b_0\eta)$ and $\Gamma_0 = b_0/2\alpha_0\tau_0^2$, and the included parameters are linked as $\Gamma_0 = |D_0|/2\tau_0^2 = R_0|A_0|^2/2$.

The GVD profile supporting SLP formation could be expressed as

$$D_2(z) = D_0 f(z)\exp\left(-2b_0\int_0^z f(\xi)d\xi\right) \quad (10)$$

where $f(z) = F(z)\exp\left(2\int_0^z \gamma(\xi)d\xi\right)$ and $F(z) = R(z)S_m(0)/R_0 S_m(z)$. In the limiting case $\gamma(z) = 0$ and $F(z) = 1$ the dispersion profile reduces to $D_2(z) = D_0\exp(-2b_0 z)$, where $D_0 < 0$, and $\alpha_0 > 0$. The duration and chirp of the secant-hyperbolic FM soliton satisfy the relation $\tau_s(z)\alpha(z) = \tau_0\alpha_0$, where

$$\tau_s(z) = \tau_0\exp\left(-2b_0\int_0^z f(\xi)d\xi\right), \quad \alpha(z) = \alpha_0\exp\left(2b_0\int_0^z f(\xi)d\xi\right) \quad (11)$$

and $b_0 = \alpha_0|D_0|$. The energy of the pulse is described by Eq. (9) is $W_s = \tau_0|A_0|^2 = |D_0|/R_0\tau_0$.

It is worth noting that modern optical fiber technology allows manufacturing of fibers with negligible losses ($K(z) = 0$) and arbitrarily controllable GVD distribution along the fiber length (for example, single-mode fibers with W-type refractive index profile) [22,23,20,24]. For such fibers the effective mode area and nonlinearity coefficient can be considered almost unchangeable over the entire fiber length making our assumption $F(z) = 1$ used for delivering of Eq. (11) justifiable.

In the case of a constant gain increment $g(z) = g_0$ the function $f(z) = \exp(2g_0 z)$ and the expressions for the GVD providing an amplification of the FM pulse and its duration take the forms:

$$D_2(z) = -|D_0|\exp\left[-\frac{\alpha_0|D_0|}{g_0}\left(\exp(2g_0 z) - 1\right) + 2g_0 z\right], \quad (12)$$

$$\tau(z) = \tau_0\exp\left[-\frac{\alpha_0|D_0|}{g_0}\left(\exp(2g_0 z) - 1\right)\right]. \quad (13)$$