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Constructing the nearly deterministic Toffoli polarization gate with the spatial degree of freedom based on weak cross-Kerr nonlinearities

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ABSTRACT

The quantum Toffoli gate can be utilized as a fundamental building block for translating more complicated classical operations into quantum algorithms, so its efficient construction is more conducive to the realization of quantum computation in large-scale than two-qubit universal quantum logic gates. Assisted by the spatial degree of freedom based on weak cross-Kerr nonlinearities, a construction scheme of the nearly deterministic Toffoli gate is presented. After the measurement on the coherent state, the required swap transformations and single-photon transformations are performed by the classical feed-forward to complete the construction task if nonzero phase shifts are displayed. The scheme proposed here is more efficient than those based on the standard model of linear optics and more feasible than those fulfilled by a variety of two-qubit universal quantum logic gates and one-qubit gates. Moreover, with mature measurement methods and simple optical elements and operations, this scheme enhances the feasibility of experiments to construct the Toffoli gate.

1. Introduction

The past several decades have witnessed the great progresses of the study of quantum logic gates in a variety of physical systems, either theoretical aspect [1–9] or experimental aspect [10–15].

As we know, a series of universal logic gates can realize any complicated computation [16], for instance, two-qubit universal quantum logic gates such as controlled-not gates and arbitrary single-qubit rotations. As for three-qubit universal quantum logic gates, a Fredkin gate and a Toffoli gate [17] can also realize any computation combined with a few one-qubit gates. Based on the three-qubit universal quantum logic gates, the multi-qubit quantum logic gates to complete complicated computation can be constructed more simply than the combination of a number of two-qubit universal quantum logic gates.

It has been showed that the Fredkin gate can be constructed by the Toffoli gate combined with two controlled-not gates, and the fault-tolerant Toffoli gate can be constructed by four T gates and Clifford-group operations [18] or five conditional two-qubit gates [19,20]. Furthermore, the construction scheme of the Toffoli gate with three global two-qubit gates was proposed [21].

Employing the standard model of linear optics [22], the quantum logic gates can be constructed with probability less than unity [23]. However, assisted by cross-Kerr nonlinear interaction, the limitation on the success probability less than unity can be broken up [24]. So numerous schemes of two-photon universal logic gates [25–29], three-photon universal logic gates and multi-photon logic gates were constructed based on cross-Kerr nonlinearities [25,26,30–34].

As mentioned above, the conventional construction schemes of the three-qubit universal quantum logic gates are complicated if they are completed by two-qubit universal quantum logic gates. In addition, the construction schemes based on the standard model of linear optics are probabilistic and the corresponding success probabilities are smaller and smaller with the increasing number of photon qubits. So cross-Kerr nonlinearities attracts a lot of attention, which can fulfill the tasks of quantum information processing with the probability near unity and need not combination of so many two-qubit universal quantum logic gates.

A Toffoli gate invented by Tommaso Toffoli is also called as the controlled–controlled-not gate, which tells its function, that is $|x,y,z\rangle \rightarrow |x,y,z\oplus xy\rangle$, where x, y, and z are binary number 0 or 1 and $\oplus$ denotes binary addition. That is, a Toffoli gate has three-qubit inputs and outputs, and if only both of the former two qubits are 1, the third qubit is inverted. The quantum Toffoli gate is commonly utilized as

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a fundamental building block for translating more complex classical operations into quantum algorithms, such as Shor's factoring algorithm and quantum simulation, by which the quantum computation in large-scale can be expected [35]. It can be said that the Toffoli gate is critically important in the field of quantum computing, and how to simply and feasibly construct the Toffoli gate is interesting and meaningful for the large-scale quantum computation [18].

In this paper, we consider the construction of a nearly deterministic Toffoli gate based on weak cross-Kerr nonlinearities, which does not need the complicated steps as the schemes fulfilled by the two-photon universal gates and single-photon gates. Moreover, it can be realized with the probability near close to unity. The mature measurement method and available operations enable the realization of this scheme to be feasible and expectable.

2. A nearly deterministic Toffoli gate assisted by weak cross-Kerr nonlinearities

In the optical systems, the polarization of photons is commonly utilized as the qubits and the horizontal mode $|H\rangle$ represents 0 and the vertical mode $|V\rangle$ represents 1.

Selected two polarization photons, $|\psi\rangle_{C_1} = (a_1|H\rangle + b_1|V\rangle)_{C_1}$, $|\psi\rangle_{C_2} = (a_2|H\rangle + b_2|V\rangle)_{C_2}$, as the control photons, and one polarization photon, which is in the state $|\phi\rangle_T$, as the target photon, and the Toffoli gate makes the following change,

$$|\psi\rangle_{C_1}|\psi\rangle_{C_2}|\phi\rangle_T \xrightarrow{\text{Toffoli gate}} (a_1a_2|HH\rangle + a_1b_2|HV\rangle + b_1a_2|VH\rangle)_{C_1C_2}|\phi\rangle_T + b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_T, \quad (1)$$

where $|\bar{\phi}\rangle_T$ is the inversion (bit-flip) on the initial state of the target photon ($|\phi\rangle_T$).

In what follows, we propose a construction of the Toffoli gate with the assistance of weak cross-Kerr nonlinearities, which can be illustrated in Fig. 1.

Step S1: Entangling the first control photon and the target photon. Passing through (PBS_1, BS_1) and the Kerr medium, photons (C_1, T) and coherent state $|\alpha_1\rangle$ evolve as

$$\begin{aligned}|\psi\rangle_{C_1}|\phi\rangle_T|\alpha_1\rangle &\xrightarrow{PBS_1,BS_1} \frac{1}{\sqrt{2}}(a_1|H\rangle_{C_{12}} + b_1|V\rangle_{C_{11}})(|\phi\rangle_{T_1} + |\phi\rangle_{T_2})|\alpha_1\rangle \\ &\xrightarrow{\text{Kerr medium}} \frac{1}{\sqrt{2}}(a_1|H\rangle_{C_{12}}|\phi\rangle_{T_1}|\alpha_1\rangle + a_1|H\rangle_{C_{12}}|\phi\rangle_{T_2}|\alpha_1 e^{-i\theta}\rangle \\ &\quad + b_1|V\rangle_{C_{11}}|\phi\rangle_{T_1}|\alpha_1 e^{i\theta}\rangle \\ &\quad + b_1|V\rangle_{C_{11}}|\phi\rangle_{T_2}|\alpha_1\rangle),\end{aligned} \quad (2)$$

where subscripts (C_{11}, C_{12}) and (T_1, T_2) stand for potential paths of photon C_1 and photon T.

To disentangle the coherent state from photons (C_1, T), an X Homodyne measurement is performed to distinguish zero phase shift and nonzero phase shifts $\pm\theta$ of the coherent state [36]. As for the X Homodyne measurement, one measurement outcome is zero phase shift and the other measurement outcomes are nonzero phase shifts θ or $-\theta$, that is

$$\begin{aligned}&\langle x|\alpha\rangle(a_1|H\rangle_{C_{12}}|\phi\rangle_{T_1} + b_1|V\rangle_{C_{11}}|\phi\rangle_{T_2}) + \langle x|\alpha e^{-i\theta}\rangle(a_1|H\rangle_{C_{12}}|\phi\rangle_{T_2}) \\ &\quad + \langle x|\alpha e^{i\theta}\rangle(b_1|V\rangle_{C_{11}}|\phi\rangle_{T_1}) \\ &= f(x,\alpha)(a_1|H\rangle_{C_{12}}|\phi\rangle_{T_1} + b_1|V\rangle_{C_{11}}|\phi\rangle_{T_2}) + f(x,\alpha\cos\theta) \\ &\quad \times (e^{-i\xi(x,\theta)}a_1|H\rangle_{C_{12}}|\phi\rangle_{T_2} + e^{i\xi(x,\theta)}b_1|V\rangle_{C_{11}}|\phi\rangle_{T_1}),\end{aligned} \quad (3)$$

where,

$$\begin{aligned}&f(x,\alpha\cos\theta) = (2\pi)^{-1/4}\exp[-(x-2\alpha\cos\theta)^2/4], \\ &f(x,\alpha) = (2\pi)^{-1/4}\exp[-(x-2\alpha)^2/4], \\ &\xi(x,\theta) = \alpha\sin\theta(x-2\alpha\cos\theta).\end{aligned} \quad (4)$$

If the zero phase shift happens, the state of photons (C_1, T) is

$$a_1|H\rangle_{C_{12}}|\phi\rangle_{T_1} + b_1|V\rangle_{C_{11}}|\phi\rangle_{T_2}. \quad (5)$$

Otherwise, if measurement outcomes imply nonzero phase shifts $\pm\theta$, the state of photons (C_1, T) is

$$e^{-i\xi(x,\theta)}a_1|H\rangle_{C_{12}}|\phi\rangle_{T_2} + e^{i\xi(x,\theta)}b_1|V\rangle_{C_{11}}|\phi\rangle_{T_1}. \quad (6)$$

When zero phase occurs (shown in Eq. (5)), no operation is required. Otherwise, when nonzero phase shifts are witnessed (shown in Eq. (6)), the phase modulation [PS $2\xi(x,\theta)$] should be performed on photon C_1 passing through path C_{12}. In addition, a swap transformation (Swap Module in Fig. 1) is needed to be performed on photon T. In a word, the phase modulation and the swap transformation change the state shown in Eq. (6) to the state shown in Eq. (5).

Step S2: Dividing paths of the target photon. Passing through (BS_2, BS_3), photon T enters into four potential paths, and the system state can be denoted as

$$a_1|H\rangle_{C_1}(|\phi\rangle_{T_{11}} + |\phi\rangle_{T_{12}}) + b_1|V\rangle_{C_1}(|\phi\rangle_{T_{21}} + |\phi\rangle_{T_{22}}), \quad (7)$$

where the subscripts (T_{11}, T_{12}, T_{21}, T_{22}) stand for four potential paths of photon T. As for photon C_1, after it passes through PBS_2, path C_{11} and path C_{12} merge into path C_1, so the subscripts both C_{11} and C_{12} are changed to C_1 in Eq. (7) and later.

Step S3: Entangling the second control photon and the target photon. The procedure that photon C_2 entangles with photon T is similar to Step S1. There are three paths passing through the Kerr medium, so this step is more complicated than Step S1. If photon C_2 and photon T pass through path C_{21} and paths (T_{11}, T_{21}), coherent state $|\alpha_2\rangle$ will accumulate up phase shift θ and phase shifts (θ, θ) respectively, as illustrated in Fig. 1.

To disentangle the coherent state from photons (C_1, C_2, T), another X Homodyne measurement is performed. Two potential swap transformations and phase modulation are executed on photon T conditioned on the measurement outcomes. Passing through PBS_4, photon C_2 enters into path C_2 from path C_{21} and path C_{22}.

As a result, the system of photons (C_1, C_2, and T) evolves into

$$\begin{aligned}&a_1a_2|HH\rangle_{C_1C_2}|\phi\rangle_{T_{11}} + a_1b_2|HV\rangle_{C_1C_2}|\phi\rangle_{T_{12}} + b_1a_2|VH\rangle_{C_1C_2}|\phi\rangle_{T_{21}} \\ &\quad + b_1b_2|VV\rangle_{C_1C_2}|\phi\rangle_{T_{22}}.\end{aligned} \quad (8)$$

Step S4: Not gate transformation. A not gate transformation fulfilled by HWP 45° on path T_{22} is performed, so the following state can be obtained,

$$\begin{aligned}&a_1a_2|HH\rangle_{C_1C_2}|\phi\rangle_{T_{11}} + a_1b_2|HV\rangle_{C_1C_2}|\phi\rangle_{T_{12}} + b_1a_2|VH\rangle_{C_1C_2}|\phi\rangle_{T_{21}} \\ &\quad + b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_{T_{22}}.\end{aligned} \quad (9)$$

Step S5: Four beam splitter superposition. Passing through BS_4, BS_5, BS_6 and BS_7, photon T undergoes the superposition effect and exits from one of four potential output ports T'_{11}, T'_{12}, T'_{21} and T'_{22}, and consequently the state of three photons (C_1, C_2 and T) is

$$\begin{aligned}&(a_1a_2|HH\rangle + a_1b_2|HV\rangle + b_1a_2|VH\rangle)_{C_1C_2}|\phi\rangle_{T'_{11}} + b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_{T'_{11}} \\ &\quad + (a_1a_2|HH\rangle + a_1b_2|HV\rangle - b_1a_2|VH\rangle)_{C_1C_2}|\phi\rangle_{T'_{12}} \\ &\quad - b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_{T'_{12}} \\ &\quad + (a_1a_2|HH\rangle - a_1b_2|HV\rangle + b_1a_2|VH\rangle)_{C_1C_2}|\phi\rangle_{T'_{21}} \\ &\quad - b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_{T'_{21}} \\ &\quad + (a_1a_2|HH\rangle - a_1b_2|HV\rangle - b_1a_2|VH\rangle)_{C_1C_2}|\phi\rangle_{T'_{22}} \\ &\quad + b_1b_2|VV\rangle_{C_1C_2}|\bar{\phi}\rangle_{T'_{22}}.\end{aligned} \quad (10)$$

Step S6: Detecting output ports of the target photon. To obtain the function of a Toffoli gate, four nondemolition detectors are placed into four terminals of photon T. From Eq. (10), it can be seen that if photon T is detected at the output port T'_{11}, no extra operation will be needed, and the target state shown in Eq. (1) is obtained. But if any one of other three detectors responds, phase-flip transformation (Z) should be performed on photons (C_1 or/and C_2) to fulfill the Toffoli gate. Explicitly, the