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Implementing arbitrary coined two-dimensional quantum walks via bulk optical interferometry

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ABSTRACT

Multi-dimensional quantum walks provide a powerful tool for simulating quantum phenomena. We design a feasible scheme to implement two-dimensional quantum walks in a "real" position space, demonstrating a scalable quantum walk on a non-trivial graph structure with single photons and bulk optical interferometry. By combining the spatial modes and polarizations of photons, we expand the dimensions of the coin states from two to four and implement arbitrary four-side coin flipping. Furthermore, with the growth of the number of walk steps, the number of linear optical elements increases linearly. This significantly reduces the resources necessary for its feasible experimental realization. Our scheme is then remarkably scalable and feasible with current technologies. Our results illustrate the potential of a two-dimensional quantum walk as a route for simulating and understanding complex quantum systems.

1. Introduction

A Quantum walk (QW) [1–4] serves as an ideal test-bed for studying the dynamics of quantum systems. In the field of quantum simulation, QWs are emerging as a versatile tool. Especially, multi-dimensional QWs can exhibit highly non-trivial topological structure, providing a powerful tool for simulating topological phenomena [5–11]. Besides, QWs have attracted attention also due to their applications such as developing new quantum algorithms [12–15] and transferring quantum states [16,17]. A one-dimensional (1D) QW is the most studied example and has been demonstrated in a number of physical systems, such as nuclear magnetic resonance [18], trapped atoms and ions [19–23], linear optics [24–29] and integrated optics [30–32]. While theoretical investigations already take advantage of complex graph structures in higher dimensions [33–36], experimental implementations are still limited by the required physical resources.

The optical approaches to increasing the complexity of a linear QW showed that the dimensionality of the system is effectively expanded by using two walkers, keeping the lattice one-dimensional [31,37,38]. While adding additional walkers to the system is promising, introducing conditioned interactions and in particular controlled non-linear interactions at the single photon level is technologically very challenging. In this paper, instead of two-walker in 1D QW, we propose a feasible scheme to implement the "real" 2D QW in position space with a single walker via linear optical elements. By combining the spatial modes and polarizations of photons, we expand the dimensions of the coin states from two to four and implement arbitrary four-side coin. The photons propagate along transverse and longitudinal directions based on both of their polarizations and positions. Considering the present technologies of linear optics, we can implement arbitrary 2D coin rotation and our scheme of 2D QWs is highly scalable.

In our proposal, QWs are implemented by shifting the position of photons in their spatial modes, compared to other realizations which have employed "abstract" position spaces, such as the time domain with circulating light pulse, or the phase space with trapped ions, or the transverse modes of the beam [39–41]. "Real" position space here means that the walkers (photons) propagates in their spatial modes instead of "abstract" position space. In the experiment in Ref. [41], a discrete-time QW taking place in the orbital angular momentum space of light, both for a single photon and for two simultaneous photons. Their experimental realization of QWs is different compared to our proposal, First, the whole process develops in a single light beam and there is no interferometer. Second, instead of "real" position space, they employ the "abstract" position space, i.e., the transverse modes of the beam. Third, it is challenging to realize inhomogeneous QWs in their experimental setup.

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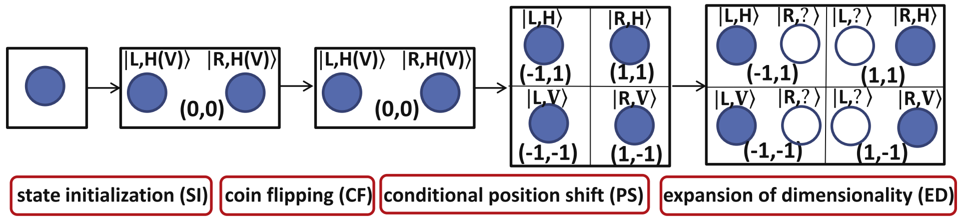

Fig. 1. Implementation of the first step of a 2D QW, involving state initialization (SI), coin flipping (CP), conditional position shift (PS) and expansion of dimensionality (ED). The hollow circles indicate the states appears during the expansion stage. Their coefficients are zero for the first step and will be filled in after the coin flipping for the next step.

The paper is organized as followings. First, we give a brief review of 1D and 2D QWs. Then, we propose a feasible scheme to implement 2D QW via bulk optical interferometry. Finally, we summarize and discuss the advantages of our scheme.

2. Quantum walks in two dimensions

To define a 1D QW, we need a 2D coin registering with two states, one for each direction: $|0\rangle$ and $|1\rangle$. The locations of the walker on the 1D lattice are labeled by the x coordinate as $|x\rangle$. At each step, we perform an arbitrary single-qubit rotation $C_{1\mathrm{D}}$ on the coin state and then evolve the walker + coin system according to the state of the coin. The unitary operator of single-step of 1D QWs [42] is in the form

$$U_{1\mathrm{D}} = \sum_x \left(|x+1\rangle\langle x| \otimes |0\rangle\langle 0| + |x-1\rangle\langle x| \otimes |1\rangle\langle 1|\right) \times \left(\mathbb{I}_{\mathrm{w}} \otimes C_{1\mathrm{D}}\right), \tag{1}$$

where $\mathbb{I}_{\mathrm{w}}$ is the identity matrix for the walker. Assume the initial state of the walker-coin system as $|\psi_0\rangle$. The final state of the system after t steps evolves to $|\psi_t\rangle = \left(U_{1\mathrm{D}}\right)^t |\psi_0\rangle$.

In the 1D QW, the walker walks on a line, while for the 2D QW, the locations of the walker are on the 2D lattice and labeled by their x and y coordinate as $|x, y\rangle$. Coins register with four states, one for each direction: $|+-\rangle$, $|++\rangle$, $|--\rangle$ and $|-+\rangle$, where $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$. For the coin state $|i, j\rangle$ $(i, j = +, -)$, the walker state $|x, y\rangle$ evolves to $|x+i, y+j\rangle$. The evolution of system is governed by two unitary operators: conditional shift operator and coin flipping operator. The whole unitary operator of evolution can be rewritten as

$$U_{2\mathrm{D}} = S\left(\mathbb{I}_x \otimes \mathbb{I}_y \otimes C_{2\mathrm{D}}\right), \tag{2}$$

where the 2D coin operator $C_{2\mathrm{D}}$ is an arbitrary two-qubit rotation and the conditional position shift operator of the 2D QW is

$$\begin{aligned} S = \sum_x \sum_y (&|x+1, y+1\rangle\langle x, y| \otimes |+,+\rangle\langle +,+| \\ &+ |x+1, y-1\rangle\langle x, y| \otimes |+,-\rangle\langle +,-| \\ &+ |x-1, y+1\rangle\langle x, y| \otimes |-,+\rangle\langle -,+| \\ &+ |x-1, y-1\rangle\langle x, y| \otimes |-,-\rangle\langle -,-|). \end{aligned} \tag{3}$$

Coin operator $C_{2\mathrm{D}}$ as an arbitrary two-qubit unitary transformation can be decomposed using the "cosine-sine" (CS) decomposition [43]:

$$C_{2\mathrm{D}} = \begin{pmatrix} L_1^\dagger & 0 \\ 0 & L_2^\dagger \end{pmatrix} U^{(4)} \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}, \tag{4}$$

where

$$U^{(4)} = \left(\begin{array}{cc|cc} \cos\theta_1 & 0 & -\sin\theta_1 & 0 \\ 0 & \cos\theta_2 & 0 & -\sin\theta_2 \\ \hline \sin\theta_1 & 0 & \cos\theta_1 & 0 \\ 0 & \sin\theta_2 & 0 & \cos\theta_2 \end{array}\right), \tag{5}$$

and $L_1^\dagger$, $L_2^\dagger$, R_1 and R_2 are single-qubit rotations.

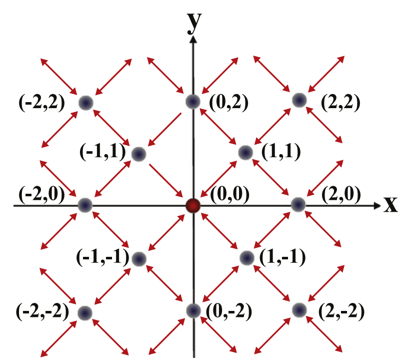

Fig. 2. The 2D lattice of vertices that represent the state space of two walkers populating a position lattice in an interferometer network.

According to this decomposition, we use Grover coin as an example. The Grover coin C_G takes the form as

$$C_G = \frac{1}{2}\begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}, \tag{6}$$

and can be rewritten as

$$C_G = \begin{pmatrix} -H & 0 \\ 0 & H \end{pmatrix} U_G \begin{pmatrix} H & 0 \\ 0 & \sigma_z H \end{pmatrix}, \tag{7}$$

where

$$U_G = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{8}$$

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ is Hadamard operator, and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is one of the Pauli operators.

This decomposition is especially useful to construct an arbitrary coin operator for 2D QWs with linear optics.

3. Implementation of 2D quantum walks with linear optics

In optical implementation of 1D QWs, we use single photon as walker who simultaneously moves in both directions in every position, and two polarization states $\{|H\rangle, |V\rangle\}$ of single photon can be used to implement two orthogonal coin states. Whereas, for the 2D QW, however, the walker simultaneously moves towards all the four directions. So we need a 4D Hilbert space for the coin states. Since single photon has only two polarization states, in 2D case, we combine two polarizations of a photon and two possible spatial modes—left (L) and right (R), to present four coin states. Therefore, four coin states $|+-\rangle$, $|++\rangle$, $|--\rangle$, $|-+\rangle$, can be represented by $|RH\rangle$, $|RV\rangle$, $|LH\rangle$, $|LV\rangle$ of single photon with two polarizations in two possible spatial modes, respectively.