Optics Communications 424 (2018) 26–31

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Coherence and entanglement dynamics of vibrating qubits

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ARTICLE INFO

Keywords:
Qubits
Coherence
Entanglement
Cavity modes
Vibrational modes

ABSTRACT

We investigate the dynamics of coherence and entanglement of vibrating qubits. Firstly, we consider a single trapped ion qubit inside a perfect cavity and successively we use it to construct a bipartite system made of two of such subsystems, taken identical and noninteracting. As a general result, we find that qubit vibration can lead to prolonging initial coherence in both single-qubit and two-qubit system. However, despite of this coherence preservation, we show that the decay of the entanglement between the two qubits is sped up by the vibrational motion of the qubits. Furthermore, we highlight how the dynamics of photon–phonon correlations between cavity mode and vibrational mode, which may serve as a further useful resource stored in the single-qubit system, is strongly affected by the initial state of the qubit. These results provide new insights about the ability of systems made of moving qubits in maintaining quantum resources compared to systems of stationary qubits.

1. Introduction

Quantum coherence and entanglement are the two most significant features of quantum theory which emerge due to the superposition principle [1–5]. Generally, a system consisting of two or more subsystems is said to be entangled if its quantum state cannot be described as a simple product of the quantum states of the constituent subsystems, which means that the state is not separable. Nowadays, it has been recognized that quantum entanglement is an essential tool for quantum information processes, such as quantum teleportation [6], quantum error correction [7,8], quantum cryptography [9] and quantum dense coding [10]. On the other hand, quantum coherence is more fundamental than entanglement. Quantum coherence not only exists in multipartite systems but also in single-partite systems. Recent studies suggest that quantum coherence can be employed as a resource, similarly to entanglement, in various quantum information tasks [1,11–15]. Several proposals have been recently put forward to define valid measures for quantifying coherence in quantum systems [1,11,16–19].

Atom–photon interactions provide a convenient way to generate and manipulate quantum coherence and entanglement. The simplest situation of atom–photon interaction is that of a two-level atom inside a cavity sustaining a single electromagnetic field mode, described by the famous Jaynes–Cummings Hamiltonian [20]. In this context, it is well known that the coupling of an atom to the cavity field is position-dependent, which in turn makes the atom-field coupling for a moving atom qubit time dependent [21,22]. Recent developments in cavity quantum electrodynamics (QED) setups offer the possibility to trap an ion inside a cavity [23,24]. In a Paul trap system, the trapping potential can be approximated to be harmonic. Hence, the center-of-mass motion of an ion in such a trap behaves as a standard harmonic oscillator. It was shown that, in the Lamb–Dicke regime, the quantized harmonic center-of-mass motion of a single two-level ion, similar to the Jaynes–Cummings model (JCM), can be coupled to its own internal electronic states while the ion is interacting with a classical single-mode traveling field [25]. This model has been then utilized for a single two-level trapped ion inside a single-mode high-Q cavity [26], demonstrating that the interaction of a cavity quantized mode with the trapped ion, within the Lamb–Dicke approximation, can lead to the generation of Greenberger–Horne–Zeilinger states. Owing to the analogy between an ion vibrating in a trapping potential and an ion interacting with a quantized cavity field, many effects and ideas observed in the context of cavity QED, such as quantum state engineering [27–33], quantum computing [34,35] and quantum state endoscopy [36,37] can be extended to the trapped ion models. Moreover, the physics of trapped ions has allowed researchers to propose some schemes

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https://doi.org/10.1016/j.optcom.2018.04.044
Received 1 November 2017; Accepted 16 April 2018

for generating entanglement. For instance, a scheme for generating a phonon–photon Bell-type state has been introduced [38] and the time behavior of entanglement between cavity mode and vibrational mode (mode–mode entanglement) for a system of two trapped ions inside a leaky cavity has been investigated [39]. A further study has been carried out concerning mode–mode entanglement in a system made of a cluster of N trapped ions interacting dispersively with a quantized electromagnetic field [40].

In spite of these studies, the evolution of coherence and mode–mode correlations of a single trapped ion qubit has not been investigated so far. Furthermore, entanglement and coherence dynamics of independent trapped vibrating qubits inside separated cavities has remained unexplored. Since separated qubit subsystems represent one of the preferred scenarios for quantum networks, the evolution of such systems deserves a dedicated case study. Motivated by these considerations, we first focus on a system containing a trapped single two-level ion (qubit) inside a high-Q cavity which is interacting with its vibrational degrees of freedom and cavity modes. Subsequently, we extend our analysis to a system composed of two of such subsystems, which are separated and initially entangled. We strive to comprehend how the center-of-mass motion of the qubits influences the dynamics of coherence and entanglement. We also examine the effect of qubit initial state, intensity of both cavity and vibrational modes on the mode–mode correlation.

The paper is organized as follows. In Section 2, we present the results about the effect of qubit, cavity and vibrational parameters on the dynamics of ion qubit coherence and mode–mode correlation. In Section 3, we extend the study to the bipartite system, analyzing the evolution of entanglement and coherence between two separated identical trapped qubits for different values of the parameters. In Section 4 we give our conclusions.

2. Single-qubit system

The system under investigation is a single two-level ion (qubit) trapped in a linear Paul trap and located inside a single-mode high-Q cavity. Owing to the confinement of the qubit in the Paul trap, the qubit vibrates with a high frequency comparable to or larger than the fundamental frequency of the cavity field. We assume that the trap axis coincides with the axis of the cavity so that, as already discussed [26,38], the internal states of the qubit (namely, the excited state $|e\rangle$ and the ground state $|g\rangle$) are coupled to both the cavity field (cavity mode) and the vibrational degrees of freedom (vibrational mode). The Hamiltonian corresponding to such a system is given by

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{H}_{int},\\ \hat{H}_0 &= \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_{\rm v}\left(\hat{a}^\dagger\hat{a}+\frac{1}{2}\right) + \hbar\omega\left(\hat{b}^\dagger\hat{b}+\frac{1}{2}\right),\\ \hat{H}_{int} &= \hbar\kappa\sin[\eta(\hat{b}+\hat{b}^\dagger)](\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{a}+\hat{a}^\dagger),\end{aligned} \tag{1}$$

where $\hat{a}^\dagger(\hat{a})$ is the creation (annihilation) operator for the cavity mode with frequency ω, $\hat{b}^\dagger(\hat{b})$ denotes the creation (annihilation) operator of the center-of-mass vibrational motion of the qubit with frequency $\omega_{\rm v}$, $\hat{\sigma}_+ = |e\rangle\langle g|$, $\hat{\sigma}_- = |g\rangle\langle e|$ and $\hat{\sigma}_z = |e\rangle\langle e| - |g\rangle\langle g|$ are the qubit operators, κ is the coupling constant between cavity mode and qubit. Moreover, η represents the Lamb–Dicke parameter.

We suppose the trapped qubit is constrained in the Lamb–Dicke regime, and the Lamb–Dicke parameter meets the condition $\eta \ll 1$. In this regime, $\hat{H}_{int}$ can be approximated by the expansion to the first order in η as

$$\hat{H}_{int} = \hbar\kappa\eta(\hat{a}+\hat{a}^\dagger)(\hat{\sigma}_+ + \hat{\sigma}_-)(\hat{b}+\hat{b}^\dagger). \tag{2}$$

In the following, we investigate a special case in which the cavity field is tuned to the first red sideband: $\omega_0 - \omega = \omega_{\rm v}$. Under this condition, by dropping the rapidly oscillating terms, the interaction picture Hamiltonian becomes

$$\hat{H}_I^r = \hbar\kappa\eta(\hat{\sigma}_+\hat{a}\hat{b} + \hat{\sigma}_-\hat{a}^\dagger\hat{b}^\dagger). \tag{3}$$

Let us take the system initially in a product state with the ion qubit in a coherent superposition of its internal states $|\psi\rangle = C_e|e\rangle + C_g|g\rangle$ ($|C_e|^2 + |C_g|^2 = 1$) while the qubit center-of-mass motion and the cavity field are, respectively, in the coherent states $|\alpha\rangle = \sum_m w_m|m\rangle$ and $|\beta\rangle = \sum_n w_n|n\rangle$, where $|m\rangle$, $|n\rangle$ are the excitation number (Fock) states while $w_m = e^{-|\alpha|^2/2}|\alpha|^m/\sqrt{m!}$ and $w_n = e^{-|\beta|^2/2}|\beta|^n/\sqrt{n!}$ denote the coherent distribution of the number states: $|\alpha|^2$ is the mean value of the photon number in the cavity field and $|\beta|^2$ is the mean value of the phonon number. The overall initial state is thus

$$|\Psi_{\rm tot}(0)\rangle = \sum_m w_m|m\rangle \otimes \sum_n w_n|n\rangle \otimes (C_e|e\rangle + C_g|g\rangle), \tag{4}$$

so that, at any later time, the state vector of the system can be written as

$$\begin{aligned}|\Psi_{\rm tot}(t)\rangle = \sum_{m,n}\{&[C_eA_{m,n}(t) + C_gB_{m,n}(t)]|m\rangle|n\rangle|e\rangle\\ &+[C_gC_{m,n}(t) + C_eD_{m,n}(t)]|m\rangle|n\rangle|g\rangle\}\end{aligned} \tag{5}$$

The time-dependent coefficients $A_{m,n}(t)$, $B_{m,n}(t)$, $C_{m,n}(t)$ and $D_{m,n}(t)$ can be found by substituting Eq. (5) into the Schrödinger equation $i\hbar\partial|\Psi_{\rm tot}(t)\rangle/\partial t = \hat{H}_I^r|\Psi_{\rm tot}(t)\rangle$ that gives

$$\begin{aligned}A_{m,n}(t) &= w_mw_n\cos[\eta\kappa t\sqrt{(m+1)(n+1)}],\\ B_{m,n}(t) &= -iw_{m+1}w_{n+1}\sin[\eta\kappa t\sqrt{(m+1)(n+1)}],\\ C_{m,n}(t) &= w_mw_n\cos[\eta\kappa t\sqrt{mn}],\\ D_{m,n}(t) &= -iw_mw_n\sin[\eta\kappa t\sqrt{mn}].\end{aligned} \tag{6}$$

Taking the partial trace of the global density matrix $\rho_{\rm tot}(t) = |\Psi_{\rm tot}(t)\rangle\langle\Psi_{\rm tot}(t)|$ over the cavity field and vibrational mode degrees of freedom, the reduced density matrix of the qubit in the basis $\{|e\rangle, |g\rangle\}$ results to be

$$\rho_{\rm q}(t) = \begin{pmatrix}\rho_{ee}(t) & \rho_{eg}(t)\\ \rho_{ge}(t) & \rho_{gg}(t)\end{pmatrix}, \tag{7}$$

where

$$\begin{aligned}\rho_{ee}(t) &= \sum_{m,n}|C_eA_{m,n}(t) + C_gB_{m,n}(t)|^2,\\ \rho_{gg}(t) &= \sum_{m,n}|C_gC_{m,n}(t) + C_eD_{m,n}(t)|^2 = 1 - \rho_{ee}(t),\\ \rho_{eg}(t) &= \sum_{m,n}[(C_eA_{m,n}(t) + C_gB_{m,n}(t))\\ &\quad\times(C_gC_{m,n}(t) + C_eD_{m,n}(t))^*] = \rho_{ge}^*(t).\end{aligned} \tag{8}$$

2.1. Coherence dynamics of the qubit

We now study the effect of cavity and vibrational parameters on the time evolution of coherence in our qubit system. Many bona-fide quantifiers of quantum coherence have been introduced [1]. Among these quantifiers, we adopt an intuitive measure which relies on the off-diagonal elements of the target quantum state which is defined by [12]

$$\zeta(t) = \sum_{i,j\ (i\neq j)}|\rho_{ij}(t)|, \tag{9}$$

where $\rho_{ij}(t)$ ($i \neq j$) are the off-diagonal elements of the system density matrix $\rho(t)$.

We easily obtain the single-qubit coherence evolution by using $\zeta(t)$ with the density matrix $\rho_{\rm q}(t)$ of Eq. (7). Fig. 1 illustrates the effect of intensity of cavity and vibrational modes on the time evolution of coherence starting from a maximally coherent state ($\zeta(0) = 1$) in the basis $\{|e\rangle, |g\rangle\}$, for a stationary qubit (column I) and vibrating qubit (column II). As can be seen when the qubit is motionless, increasing the intensity of the cavity field (larger mean photon number) preserves the initial coherence for longer times. However, coherence preservation is more effective for vibrating qubits. In fact, as displayed in the plots of