



The study of pixelated sampling and its influences on the 2nd-order spatial coherence measurement

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ABSTRACT

Since the measurement of spatial coherence requires multiple sampling to acquire the complex coherence factor of a wavefield, it is convenient to accomplish such dynamic work by a non-redundant multi-slit through a programmable device such as spatial light modulator (SLM). In this study, we examined the influence of SLM-like pixelated structure, parameterized by whose fill factor, on both intensity distribution (first-order statistics) and complex coherence factor (second-order statistics). Based on the generalized Van Cittert–Zernike theorem that as the source satisfied the quasi-homogeneous condition, the coherence factor of the wavefield was irrelevant to the pixelated sampling structure. The inference was validated by a SLM-based experiment from a pseudothermal light source. We expected this work to be useful for future dynamic coherence measurement.

1. Introduction

Wavefield correlation has attracted much attention not only its characteristic plays an essential role in interference and diffraction physics but also opens the window for the unconventional imaging [1–3]. Recently, Kondakci et al. [4] and El-Halawany [5] achieved a far-field lensless identification by measuring the complex coherence factor of the light scattering off an obstructive object. Among such remarkable works, one important issue lies in how to retrieve the statistical properties of a wavefield through the sampling process.

Typically, the complex coherence factor of a wavefield could be obtained by measuring the fringe visibility of the interference pattern based on the Young's double-slit experiment [6]. Afterward, a number of optical masks rather than Young's double-slit were proposed with corresponding signal analyses for the coherence measurement, such as the two-dimensional interferogram analysis [7–11], marginal power spectrum approach [12–15], redundant and non-redundant arrays [16–21]. No matter what signal was analyzed to retrieve the statistical behavior of the wavefield, in terms of efficiency and convenience of experiment, nowadays apparatus was developed toward the dynamic sampling via the programmable control like spatial light modulators (SLMs), digital micromirror devices (DMDs) or liquid crystal displays (LCDs) [22–26]. Obviously, the pixelated structure will definitely influence the diffraction, resulting in the different intensity distribution (usually called the first-order coherence property in statistical optics) in comparison with the conventional continuous slits. However, to our knowledge, no work has discussed whether these pixelated devices affect the results of second-order measurement or not.

Therefore, in this study, we were asking ourselves that what impact of the pixelated structure in current dynamic devices upon the second-order of the wavefield. A series of theoretical and numerical analysis were conducted. It was found that, although different shapes of sampling mask (continuous or pixelated masks) did affect the first-order statistics of a wavefield, the second-order property would only depend on the source as if the wavefield obeyed the quasi-homogeneous and Schell-model assumption, which were consistent in most of practical cases. The theoretical analysis as well as numerical results were examined by a SLM-like experiment, where the incoherent source was implemented by passing laser light through a moving diffuser as the pseudothermal light source.

This paper is organized as follows. The analysis of pixelated and continuous structures based on the generalized Van Cittert–Zernike theorem were conducted in Section 2. The wavefield intensity were addressed in both space and spatial-frequency domain. The numerical simulation was described in Section 3, where a thousand random phase screens were utilized to ensure sufficient sampling points to characterize the partial coherence phenomenon of light. The experimental results were provided in Section 4. Finally, the validation of pixelated sampling was discussed in Section 5.

2. Theory

In this section, we reviewed the generalized Van Cittert–Zernike theorem as the beginning. With pixelated and continuous mask in sampling process, we analyzed both the first-order and second-order properties in space and spatial-frequency domains, respectively.

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2.1. The generalized Van Cittert–Zernike theorem

According to the generalized Van Cittert–Zernike theorem, the mutual intensity function \mathbf{J} of an incoherent and quasi-monochromatic source $I(\xi, \eta)$ at the observation plane could be derived by the Fourier transform of the source distribution as defined:

$$\mathbf{J}(x_1, y_1; x_2, y_2) = \frac{\kappa(\bar{x}, \bar{y}) e^{-j\varphi}}{(\bar{\lambda}z)^2} \iint_{-\infty}^{\infty} I(\bar{\xi}, \bar{\eta}) \times \exp \left[j \frac{2\pi}{\bar{\lambda}z} (\Delta x \bar{\xi} + \Delta y \bar{\eta}) \right] d\bar{\xi} d\bar{\eta}, \quad (1a)$$

$$\kappa(\bar{x}, \bar{y}) = \iint_{-\infty}^{\infty} \mu(\Delta\xi, \Delta\eta) \exp \left[j \frac{2\pi}{\bar{\lambda}z} (\bar{x} \Delta\xi + \bar{y} \Delta\eta) \right] d\Delta\xi d\Delta\eta, \quad (1b)$$

where $\mu(\Delta\xi, \Delta\eta)$ was the complex coherence factor of the light source, $\bar{\lambda}$ was the central wavelength, z was the propagation distance, (ξ, η) denoted the coordinates in the source plane, and (x, y) denoted the coordinates in the observation plane. The variables $(\bar{\xi}, \bar{\eta})$, $(\Delta\xi, \Delta\eta)$, (\bar{x}, \bar{y}) , and $(\Delta x, \Delta y)$ were defined as $\bar{\xi} = (\xi_1 + \xi_2)/2$, $\bar{\eta} = (\eta_1 + \eta_2)/2$, $\Delta\xi = \xi_2 - \xi_1$, $\Delta\eta = \eta_2 - \eta_1$, $\bar{x} = (x_1 + x_2)/2$, $\bar{y} = (y_1 + y_2)/2$, $\Delta x = x_2 - x_1$, and $\Delta y = y_2 - y_1$ [2]. According to Goodman's work, when the $\mu(\Delta\xi, \Delta\eta)$ was much narrower in the $(\Delta\xi, \Delta\eta)$ plane than the $I(\bar{\xi}, \bar{\eta})$ was in the $(\bar{\xi}, \bar{\eta})$ plane, Eq. (1) could be simplified and normalized to Eq. (2) as shown below,

$$\mu(\Delta x, \Delta y) = \frac{e^{-j\varphi} \iint_{-\infty}^{\infty} I(\bar{\xi}, \bar{\eta}) \exp \left[j \frac{2\pi}{\bar{\lambda}z} (\Delta x \bar{\xi} + \Delta y \bar{\eta}) \right] d\bar{\xi} d\bar{\eta}}{\iint_{-\infty}^{\infty} I(\bar{\xi}, \bar{\eta}) d\bar{\xi} d\bar{\eta}}, \quad (2)$$

where $\mu(\Delta x, \Delta y)$ was the complex coherence factor of the observation field. The phase factor φ represented the phase difference between two sampling points at the observation plane, usually neglected as a multiplicative constant in far-field or symmetric sampling circumstance.

There were two points that should be noted in Eq. (2). First, the complex coherence factor at the observation plane was merely a function of location difference $(\Delta x, \Delta y)$, which implied that the wavefield satisfies the Schell-model source [1,2]. Second, this equation also held for a partially coherent wavefield as long as the width of its intensity distribution $I(\bar{\xi}, \bar{\eta})$ in the $(\bar{\xi}, \bar{\eta})$ plane was much larger than the width of its complex coherence factor $\mu(\Delta\xi, \Delta\eta)$ in the $(\Delta\xi, \Delta\eta)$ plane. The condition was called quasi-homogeneous and was suitable in most practical cases since the coherence area of light source was usually much smaller than the source area, as a consequence of the reciprocal width relations for the Fourier transform [1,2].

2.2. Analysis of pixelated and continuous masks in space domain

To compare the effectiveness of pixelated and continuous masks on the measurement of complex coherence factor, for simplicity, we started from the common double-slit in one-dimensional case. Nonetheless, the derivation was applicable to any other type of masks such as redundant or non-redundant structures.

The first row of Fig. 1 showed the geometrical shapes of both pixelated (right column) and continuous (left column) double-slit with the same aperture diameter a and slit spacing d . For the pixelated slit, p represented the pixel pitch and f_p represents the fill factor of the device ($0 \leq f_p \leq 1$) so that the effective aperture in each pixel was pf_p .

With quasi-homogeneous source that the aperture size at the observation plane was much smaller than the intensity distribution. It could be found that the intensity variation of the wavefield within each aperture was insignificant so that the uniform wavefields sampled by the double-slit were expressed in Eq. (3):

$$U_c(x) = \left[\text{rect} \left(\frac{x}{a} \right) \right] \otimes \left[\delta \left(x + \frac{d}{2} \right) + \delta \left(x - \frac{d}{2} \right) \right], \quad (3a)$$

$$U_p(x) = \left[\text{rect} \left(\frac{x}{pf_p} \right) \otimes \sum_{n=1}^N \delta(x - x_n) \right] \otimes \left[\delta \left(x + \frac{d}{2} \right) + \delta \left(x - \frac{d}{2} \right) \right], \quad (3b)$$

where x represented the spatial coordinate in the slit plane (observation plane), x_n described the position vector of the n th pixel, $\delta(\cdot)$ was the Dirac's delta function, and \otimes denoted the convolution operation. The subscripts c and p of the wavefield U under measurement represented the continuous and pixelated sampling, respectively. Each slit was consisted of N pixels, here only three pixels was shown in Fig. 1 as the schematic illustration.

Since the variations of the wavefield within each aperture were insignificant, these wavefields sampled by each pixel could be approximated as equal amount. Hence, the far-field intensities at the focal plane of a 2f-system were derived in Eq. (4),

$$I_c(\bar{\lambda}fu) = \frac{I_0 a^2}{2\bar{\lambda}^2 f^2} \text{sinc}^2 \left(\frac{a}{\bar{\lambda}f} u \right) \left[\mu(d) e^{-j \frac{2\pi d}{\bar{\lambda}f} u} + \mu^*(d) e^{j \frac{2\pi d}{\bar{\lambda}f} u} \right], \quad (4a)$$

$$I_p(\bar{\lambda}fu) = \frac{I_0 p^2 f_p^2}{2\bar{\lambda}^2 f^2} \text{sinc}^2 \left(\frac{pf_p}{\bar{\lambda}f} u \right) \times \left\{ \sum_{n=1}^N 1 + \sum_{n=m+1}^N \sum_{m=1}^{N-1} \left[e^{-j \frac{2\pi p}{\bar{\lambda}f} (n-m)u} + e^{j \frac{2\pi p}{\bar{\lambda}f} (n-m)u} \right] \right\} \times \left[\mu(d) e^{-j \frac{2\pi d}{\bar{\lambda}f} u} + \mu^*(d) e^{j \frac{2\pi d}{\bar{\lambda}f} u} \right], \quad (4b)$$

where u represented the coordinate in the corresponding Fourier plane, I_0 denoted the field intensity sampled by each single-slit, and $\text{sinc}(u) = \sin(\pi u)/\pi u$. It was noticed that, due to insignificant variations within each aperture, the correlations between every two pixels were negligible so that the coefficients of every mutual terms became unity in Eq. (4b). Otherwise the coefficients of each mutual terms would be $\mu[(n-m)p]$, which arose from the second-order coherence within a single-slit.

For the continuous double-slit measurement, I_c (Eq. (4a)), the bracket was the interference fringe induced by the double-slit, enveloped by the diffraction of the single-slit. On the contrary, pixelated structure (Eq. (4b)) would broaden the diffractive envelope as well as introduce another interference signal into the interferogram. These interference signals affected the first-order properties of wavefield and could be analyzed effectively by the multi-aperture sampling as mentioned in previous literatures [18–20].

2.3. Analysis of pixelated and continuous masks in spatial-frequency domain

An alternative approach to retrieve the complex coherence factor of a wavefield could resort to its interferogram in spatial-frequency domain, as in Eq. (5), whereas the all classes of aperture pairs of the mask would appear by means of the spectral analysis.

$$\tilde{I}_c(f_u) = \frac{I_0 a}{2\bar{\lambda}f} \Lambda \left(\frac{\bar{\lambda}f}{a} f_u \right) \otimes \left[\mu(d) \delta \left(f_u - \frac{d}{\bar{\lambda}f} \right) + \mu^*(d) \delta \left(f_u + \frac{d}{\bar{\lambda}f} \right) \right], \quad (5a)$$

$$\tilde{I}_p(f_u) = \frac{I_0 p f_p}{2\bar{\lambda}f} \Lambda \left(\frac{\bar{\lambda}f}{p f_p} f_u \right) \otimes \left\{ N \delta(f_u) + \sum_{l=1}^{N-1} (N-l) \times \left[\delta \left(f_u - \frac{l p}{\bar{\lambda}f} \right) + \delta \left(f_u + \frac{l p}{\bar{\lambda}f} \right) \right] \right\} \otimes \left[\mu(d) \delta \left(f_u - \frac{d}{\bar{\lambda}f} \right) + \mu^*(d) \delta \left(f_u + \frac{d}{\bar{\lambda}f} \right) \right], \quad (5b)$$

where f_u represented the coordinate in spatial-frequency domain of the observation plane and $\Lambda(\cdot)$ denoted the Fourier transform of $\text{sinc}^2(\cdot)$.

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