Optics Communications 421 (2018) 50–55

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Polarization dynamics on optical axis

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ARTICLE INFO

Keywords:
Polarization
Vortex beam
Gouy phase
Optical singularity

ABSTRACT

In most tight focusing systems with the incident wave of unique polarization (including the radial and azimuthal polarization), the polarization along the optical (central) axis (the propagation direction) is found to be uniform. In this paper, we will show that when there is an off-axis vortex in the incident field, the polarization state varies along the central optical axis and the polarization singularities, 'C-points', 'L-points' both can exist on axis. The polarization state is demonstrated to be mainly dependent on two parameters, the initial position of off-axis vortex and the Gouy phase of the axial field. Due to the special property of the Gouy phase, we find that moving the off-axis vortex can vary the polarization without changing its handedness. Especially, it is quite interesting to find that one direction of the linear polarization at the geometrical focus exactly corresponds to one position of the off-axis vortex.

1. Introduction

Sometimes it is inevitable to have a field with an off-axis vortex, for example in generating an on-axis vortex with the misalignment [1–3]. While also in many cases the off-axis vortices are studied for their special properties. For examples they can be used to decrease the orbital angular momentum (OAM) of the beam [4,5] or to observe the rotational Doppler effect [6]. In a high numerical aperture (NA) system, the off-axis vortex was found to follow a 'zigzag' rotation trajectory along the propagation and lead to a shift of the intensity peak in the focal plane [7]. The axial field near the focus is very important in interferometry and metrology and its wavefront spacings along the central axis will influence the accuracy of the interferometer [8–10]. The polarization was found to have a strong effect on the axial wavefront spacings [10,11] and when a common (linear, circularly, radial and azimuthal) polarized Gaussian beam with a central vortex is strongly focused, it was observed that the polarization state of the field is the same along the central optical axis [10,12–16], even the polarization can have seven kinds of state on axis [11,17]. In this article, we will show that when a field with an off-axis vortex is focused strongly, the polarization state along the axis (the propagation direction) is no longer uniform and the position of the off-axis vortex has a close relation with the axial polarization state.

When the polarization state is discussed in a high NA system, usually the traditional description of the polarization in a transverse plane (which is two-dimensional, 2D) is not sufficient, since the longitudinal field component cannot be neglected any more and a three-dimensional (3D) description is necessary [10,12,13,18]. The 3D polarization properties including polarization singularities of fully polarized fields were studied in [19,20], and very recently in [21,22] the 3D polarization algebra of fully polarized and partially polarized fields has been discussed and the explicit expressions are also given for 3D complex fields in terms of Chandrasekhar–Stokes parameters. In a strongly focused, radially polarized field, the polarization can be defined in ρ–z plane [23] since there the polarization at any point can be decomposed into a radial direction (ρ) and a longitudinal direction (z), which actually is a special case of the 3D polarization description in [21]. In this article, following the same way in [23] we will use another special 3D polarization description to study the polarization dynamics on the optical central axis and we will show that the two Gouy phases and the position of the off-axis vortex work on the polarization ellipses in different ways.

2. Focusing system

Let us consider an aplanatic, high NA focusing system with a focal length f and a semiaperture angle α (see Fig. 1). In this system the origin O of a Cartesian coordinate system is located at the geometrical focus. Assume that the incident field is linearly polarized at x direction, and its complex amplitude distribution of the electric field is $V_0(r,\phi)$ with r the radial distance and ϕ the azimuthal angle. According to Richards and

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https://doi.org/10.1016/j.optcom.2018.03.080
Received 29 January 2018; Received in revised form 24 March 2018; Accepted 30 March 2018

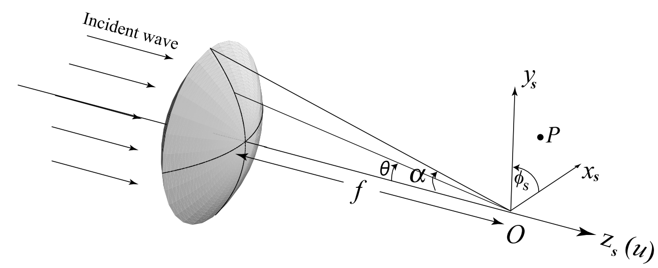

Fig. 1. Illustration of a high-numerical-aperture system.

Wolf's vector diffraction theory [12], the electric field at observation point $P(u, v, \phi_s)$ in the focal region can be expressed as

$$E(u,v,\phi_s)=\begin{bmatrix}e_x\\e_y\\e_z\end{bmatrix}=-\frac{\mathrm{i}k}{2\pi}\int_0^{\alpha}\int_0^{2\pi} fV_0(\theta,\phi)\sqrt{\cos\theta}\sin\theta \times\begin{bmatrix}\cos\theta+\sin^2\phi(1-\cos\theta)\\(\cos\theta-1)\cos\phi\sin\phi\\-\sin\theta\cos\phi\end{bmatrix}e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}e^{\mathrm{i}\frac{v\sin\theta}{\sin\alpha}\cos(\phi-\phi_s)}\mathrm{d}\phi\mathrm{d}\theta, \tag{1}$$

where u, v are the dimensionless Lommel variables [12,24], namely:

$$u=kz_s\sin^2\alpha,\qquad v=k\sqrt{x_s^2+y_s^2}\sin\alpha, \tag{2}$$

with the wave number $k = 2\pi/\lambda$ and λ is the wavelength. Here $V_0(\theta,\phi) = V_0(r,\phi)$ since $r = f\sin\theta$ because of the Abbe sine condition in this focusing system.

3. Polarization states on optical axis

Assume there is an off-axis vortex with topological charge $m = -1$ embedded in the incident Gaussian beam at position $r = a_0$, $\phi = 0$, the complex amplitude can be written as $V_0(\theta,\phi) = e^{(-f\sin\theta/\omega_0)^2}\left(f\sin\theta e^{-\mathrm{i}\phi} - a_0\right)$ with ω_0 the spot size of the beam in the waist plane (here it is also assumed that the waist plane of the Gaussian beam is coincident with the entrance plane of the focusing system). Then the total field, i.e. Eq. (1) can be calculated and the field components on optical axis ($v = 0$) can be written as [7]:

$$e_x(u)=\mathrm{i}k\frac{a_0 f}{2}\int_0^{\alpha}e^{\frac{-f^2\sin^2\theta}{\omega_0^2}}(1+\cos\theta)\sqrt{\cos\theta}\sin\theta e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}\mathrm{d}\theta, \tag{3}$$

$$e_z(u)=\mathrm{i}k\frac{f^2}{2}\int_0^{\alpha}e^{\frac{-f^2\sin^2\theta}{\omega_0^2}}\sqrt{\cos\theta}\sin^3\theta e^{\mathrm{i}\frac{u\cos\theta}{\sin^2\alpha}}\mathrm{d}\theta, \tag{4}$$

which shows that there are two nonzero field components, e_x and e_z on axis. The equations also mean that the polarization of the axial field is not uniform, which is not as same as that of the focused field with vortex in the beam center (i.e., the common focused vortex beam). It was summarized in [11] that there is only one/zero polarization state on the optical (central) axis, when a common vortex beam with any topological charge (including zero order) and any common (linear, circular, radial and azimuthal) polarization state is focused in a high NA system. Therefore it is quite interesting to see how the polarization changes along the optical axis. Before that, let us first look at the symmetry relations of the field. From Eqs. (3) and (4), we can get that

$$e_i^*(u)=-e_i(-u),\quad (i=x,z) \tag{5}$$

which indicates that

$$|e_i(u)|=|e_i(-u)|, \tag{6}$$

$$-\arg[e_i(u)]=\pi+\arg[e_i(-u)],\quad (\mathrm{mod}\ \ 2\pi) \tag{7}$$

here $\arg[e_i(u)] (i = x, z)$ denotes the phase of e_i.

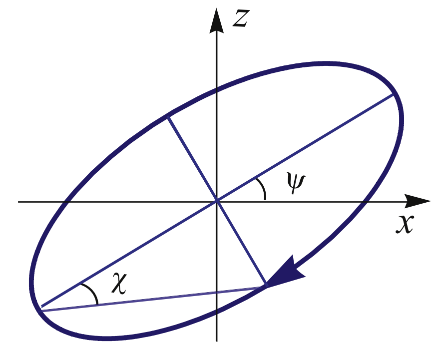

Fig. 2. Defining the angles ψ and χ of a polarization ellipse.

The Gouy phase δ is defined as the phase difference between the actual (diffracted) field and a (non-diffracted) spherical wave converging to the focus in the half-space $z_s < 0$ and diverging from it in the other half-space $z_s > 0$ [see [24], Sec.8.8.4]. For each field component on axis therefore a Gouy phase can be defined as [7]

$$\delta_i(u)=\arg[e_i(u)]-kz_s\quad (i=x,z). \tag{8}$$

According to the relation of u and z_s, Eq. (2), one can obtain

$$kz_s=u/\sin^2\alpha. \tag{9}$$

From Eqs. (7)–(9), we find that the Gouy phase satisfies

$$-\delta_i(u)=\pi+\delta_i(-u)\quad (\mathrm{mod}\ \ 2\pi),\quad (i=x,z) \tag{10}$$

and at the geometrical focus

$$\delta_i(0)=\pi/2\quad (\mathrm{mod}\ \ 2\pi),\quad (i=x,z). \tag{11}$$

The polarization of a field is usually characterized by the four Stokes parameters or two angular parameters of polarization ellipse [24, Sec. 1.4,]. These parameters in a standard description are defined in terms of e_x and e_y components when the field is propagating along the z axis. It means that the field is totally polarized in a transverse plane (or in 2D field). However, in a high NA system, the e_z component cannot be neglected any more, and the polarization should be treated in three dimension [19–23], which indicates that for any point of the field, its polarization plane can be different from others. In our case, since the field is always polarized in x–z plane, which is a special case of the general 3D polarization and actually this plane is the 'T surface' (the plane with the propagation direction) in [20]. Those polarization parameters then naturally can be described in x–z plane, which is analogous to the re-definition of the polarization plane in a focused, radially polarized field [23]. Here we can define:

$$S_0=|e_x|^2+|e_z|^2, \tag{12}$$

$$S_1=|e_x|^2-|e_z|^2, \tag{13}$$

$$S_2=2|e_x||e_z|\cos\Delta, \tag{14}$$

$$S_3=2|e_x||e_z|\sin\Delta, \tag{15}$$

where $\Delta = \arg[e_x] - \arg[e_z] = \delta_x - \delta_z$. The normalized Stokes parameters are $s_1 = S_1/S_0$, $s_2 = S_2/S_0$ and $s_3 = S_3/S_0$ which represents a point on the Poincaré sphere [24, Sec. 1.4] [25, Sec. 10.3]. The two angular parameters of the polarization ellipses are the orientation angle $\psi (0 \le \psi < \pi)$ and the ellipticity angle $\chi (-\pi/4 \le \chi \le \pi/4)$ (see Fig. 2). Here ψ is the angle between the x axis and the major axis of the polarization ellipse. $|\tan\chi|$ represents the ratio of the two axes of the ellipse and the sign of χ distinguishes the two senses of the handedness of the ellipse. These two angular parameters can be expressed in terms