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Periodic abruptly autofocusing and autodefocusing behavior of circular Airy beams in parabolic optical potentials

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ABSTRACT

In this paper, we investigate the propagation dynamics of circular Airy beams (CAB) in a medium with radial parabolic potential both analytically and numerically. Our results show that CAB will exhibit periodic abruptly autofocusing behavior under the action of radial parabolic potentials. Within every period, we interestingly observe a novel autodefocusing behavior, which manifests that optical peak intensity abruptly decreases by several orders of magnitude. In particular, such unique "autofocusing-to-autodefocusing" effect can lead to the formation of a series of elegant periodic optical bottles having paraboloidal shapes during the whole propagation process. Our findings indicate that CAB is expected to provide a unique tool in optical micromanipulation and optical trapping.

1. Introduction

During the past few years, the researches of novel CAB have attracted a great deal of attention due to its intriguing abruptly autofocusing characteristics even without invoking nonlinear effects. In 2010, the concept of such beam has been introduced theoretically by Efremidis et al. [1] and then further confirmed experimentally by Papazoglou et al. in the field of optics [2]. Because of the unique properties, such novel CAB recently finds numerous unprecedented potential applications in optical micromanipulation [3,4], generation of spatiotemporal light bullet [5] and formation of optical filament [6]. Therefore, the propagation and manipulation of CAB have recently stirred widespread interest in the scientific community. For instance, a variety of approaches including an annular aperture [7], an apodization mask [8] or a modulation phase factor [9–11] have been proposed to enhance/suppress focus intensity, or engineer the accelerated path/focus pattern, To control the propagation dynamical behaviors of CAB, some other physical mechanisms such as optical vortices [12–15], nonparaxial effect [16] and optical lattices [17] have also been introduced by some researchers both experimentally and theoretically. Very recently, we broadened the conventional family of CAB through imposing other physical parameters such as a radial chirp and a cone angle, showing that the autofocusing effect of beams can be remarkably enhanced, suppressed, and even completely eliminated, depending on the initial condition of the input parameters [18,19]. Subsequently, we further constructed another new kind of CAB which is modified by a quadratic phase modulation in spectral regime. Our analysis disclosed that such modified CAB experiences a dual abruptly focusing behavior having the same focus intensity as well as size of focus spot, thus inducing the formation of a paraboloidal optical bottle [20].

Recently, the external potential as another new physical strategy is extensively employed to control the propagation of Airy beams. For instance, Efremidis et al. demonstrated that the Airy beams can travel according to any predefined trajectory by controlling different linear index gradients [21]. Liu et al. disclosed that the self-deflection of the plasmonic Airy beams can be accelerated, compensated or even reversed without compromising the self-healing characteristics in linear optical potentials [22]. Chávez-Cerda et al. further investigated the propagation of Airy beams in linear potentials both theoretically and experimentally, showing that it is possible to just reduce the performance of self-acceleration to zero value [23]. However, up to now, most of the previous researches are only confined to the case of asymmetric Airy beams. For the case of CAB, both Hwang and Zhong disclosed that both focus positions and focus lengths can be controlled by appropriately tuning linear optical potentials [24,25]. Recently, Zhang et al. further investigate the propagation behaviors of some special beams mainly including Hermite–Gauss, Laguerre–Gauss, Bessel–Gauss and finite energy Airy beams in parabolic optical potentials while the novel CAB still remain unexplored [26–28]. Meanwhile, we have known that the CAB possesses unique abruptly autofocusing characteristics even in the linear regime, when compared to the case of conventional beams [1]. Therefore, we believe that the dynamics of CAB in parabolic potentials

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will become more intriguing and interesting. Inspired by these foregoing investigations, in this paper we present a comprehensive research on the propagation of CAB in media with external parabolic potentials, focusing specifically on the unusual abruptly focusing behaviors. The rest of this paper is organized as follows. In Section 2 we will derive an approximate solution to the paraxial physical model with external parabolic potentials, and then give a brief analysis. In Section 3, we will numerically discuss dynamical behavior of CAB based on the split-step Hankel transform. Finally, a brief conclusion will be present in Section 4.

2. Analytical analysis

Let us start by considering a radially symmetric optical beam propagating in linear media with external potentials. Under the paraxial approximation, the physical model for describing the behavior of the slowly-varying envelope $E(r, z)$ of the optical electric field can be given by [21–28]

$$\frac{\partial E}{\partial z} = \frac{i}{2}\left(\frac{\partial^2 E}{\partial r^2} + \frac{1}{r}\frac{\partial E}{\partial r}\right) - iV(r)E. \tag{1}$$

The parameter z accounts for the propagation distance scaled by the Rayleigh lengths kx_0^2, and the parameter r accounts for the radial coordinate scaled by an arbitrary transverse width x_0. Here, $k = 2\pi n/\lambda$ is the optical wavenumber, and λ is the wavelength in vacuum and n is the refractive index. The first and second terms in the right-hand side of Eq. (1) account for diffraction and the external potential, respectively. Based on the refractive-index distribution of media, the corresponding external potential function $V(r)$ mainly includes linear potentials, the parabolic potentials and the spherical potentials et al. In this paper, we only consider the parabolic potential given by $V(r) = 0.5p^2r^2$ where p represents the depth of potential since it has the unique ability to induce the occurrence of perfect periodic evolution of optical beam [26–33]. Generally speaking, both strongly nonlocal media and gradient-index media are widely applied to construct such optical parabolic potential [29–33]. Therefore, the depth of potential can be controlled through tuning optical power for the former case or the distribution of refractive index for the latter case. The initial CAB in cylindrical coordinate is defined as [1]

$$E_0(r, 0) = Ai\left(r_0 - r\right)\exp\left[\alpha\left(r_0 - r\right)\right], \tag{2}$$

where $Ai(\cdot)$ denotes the well-known Airy function, r_0 denotes the input radial position of the main lobe of CAB, and the parameter α denotes the exponential truncation factor, which ensures that CAB can be realized experimentally. In this paper, the split-step Hankel transform will be employed to retrieve the accurate numerical solution [34,35]. Following the standard procedure of Hankel transform, the diffraction term in Eq. (1) is treated in term of the following integral pair

$$E(r, z) = \frac{1}{2\pi}\int_0^\infty g_0(k, 0)\exp\left(-ik^2 z/2\right)J_0(kr)\,kdk, \tag{3}$$

$$g_0(k, 0) = 2\pi\int_0^\infty E_0(r, 0)J_0(kr)\,rdr, \tag{4}$$

where the parameter k represents the radial frequency, $J_0(\cdot)$ represents the first-order Bessel function, and $g_0(k, 0)$ represents the Hankel transform of the initial field $E_0(r, 0)$.

As it is well known, the exact solution of Eq. (1) does not exist if the expression of CAB [Eq. (2)] is employed to be as the initial condition [1,7–11]. In this paper, we will first derive an approximate analytical solution to help us intuitively estimate the unusual dynamical behavior of CAB before performing numerical simulations. When the radius of main lobe r_0 is big enough, most of the optical energy is essentially located far from the central part of the beam. Basing on our recent theories [18–20], we can approximately take the large ring-Airy beam as quasi 1-D structure through neglecting the first derivative term on the right-hand side of Eq. (1). Therefore, borrowing recent results of 1-D structure asymmetric Airy beam in Ref. [26], we easily derive the 2-D structure spatial analytical expression to approximately describe the dynamical behaviors of the CAB as follows

$$|E(r, z)| = \left|\sqrt{\frac{K}{2rb}}Ai\left(\frac{K}{2b} - \frac{1}{16b^2} + i\frac{\alpha}{2b} - r_0\right)\right| \times \exp\left[\alpha\left(\frac{K}{2b} - \frac{1}{8b^2} - r_0\right)\right], \tag{5}$$

where $b = p\cot(pz)/2$ and $K = pr/\sin(pz)$. Obviously, from Eq. (5), we deduce that the CAB accelerates along the following fashion in the r–z plane

$$r = \frac{\sin 2(pz)}{4p^2\cos(pz)} - r_0\cos(pz). \tag{6}$$

Based on (6), we can further obtain a series of special positions of optical beam propagation by setting $r = 0$ as

$$z_m^{(f)} = \arctan\left(2p\sqrt{r_0}\right)/p + (m-1)\pi/p, \tag{7}$$

$$z_m^{(d)} = -\arctan\left(2p\sqrt{r_0}\right)/p + m\pi/p, \tag{8}$$

where the parameter m is a nonzero integer. Up to now, we have arrived at the central conclusions of analytical expression in this paper, being the closed-form approximation. In fact, by comparison with the latter simulation, we easily know that the parameter $z_m^{(f)}$ corresponds to the conventional autofocusing point while the parameter $z_m^{(d)}$ corresponds to the novel autodefocusing point where the central intensity abruptly decreases. Therefore, for the purpose of convenient description latter, we take the parameters $z_m^{(f)}$ and $z_m^{(d)}$ as the focus, and defocus positions, respectively. Obviously, such analytical solution allows one to intuitively capture some main physical effects and estimates the propagating behaviors with much shorter computational times required for the simulations. First, one of the most striking features is that the approximate solution is periodic function as propagation distance with the period $T = \pi/p$. Therefore, when the propagation distance increases from the first position z_1 to the second position $z_2 = mT + z_1$, from Eq. (5) we obtain $E(r, z_1) = E(r, z_2)$. More interestingly, the CAB is found to perform periodic autofocusing and autodefocusing behaviors under the action of radial parabolic potentials since both $z_m^{(f)}$ and $z_m^{(d)}$ are periodic functions. Second, from Eq. (5) we easily arrive at $E\left(r, z_m^{(c)} + l\right) = E\left(r, z_m^{(c)} - l\right)$ with $z_m^{(c)} = (m - 1/2)T$ for a given length l, implying that optical beams at $z = z_m^{(c)} + l$ and $z = z_m^{(c)} - l$ exhibit perfect symmetric behavior with respect to the central line $z_m^{(c)}$ for every period. Third, the inspection of Eqs. (7)–(8) further shows that both focus and defocus positions monotonically decrease with the increase of p. On the other hand, the former monotonically decreases with the increase of r_0 while the latter increases monotonically. Finally, it is clear to see that the period T only depends on p, but is completely independent on r_0. Here, we need to stress that the evolution equation of the CAB [Eq. (5)] is an approximate expression because we make the following approximation $\partial^2 E/\partial r^2 + \partial E/r\partial r \approx \partial^2 E/\partial r^2$ in the derivation. Therefore, the accurate numerical results based on the split-step Hankel transform will be used to check the validity of our analytical solution in the next section. In particular, It is worthwhile to remark that Eqs. (5)–(8) are only suitable to describe the propagation characteristics of the CAB far from the focusing point since the approximation used in deriving them cannot be made near focus points. Therefore, our analytical solution cannot help us estimate the abrupt change of optical intensity, which will be discussed by employing the numerical approach latter.

3. Numerical analysis

In this section, we will further explore the propagation properties of the CAB under the action of radial parabolic potential numerically by using the split-step Hankel transform [34,35]. In the following discussion, we assume $x_0 = 1$ mm, $r_0 = 8$ and $\alpha = 0.2$ throughout the paper unless otherwise specified. In addition, the input power for