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A semi-analytical approach to the study of an elastic circular cylinder confined in a cylindrical fluid domain subjected to small-amplitude transient motions

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Abstract

This paper deals with the transient motions experienced by an elastic circular cylinder in a cylindrical fluid domain initially at rest and subjected to small-amplitude imposed displacements. Three fluid models are considered, namely potential, viscous and acoustic, to cover different fluid–structure interaction regimes. They are derived here from the general compressible Navier–Stokes equations by a formal perturbation method so as to underline their links and ranges of validity *a priori*. The resulting fluid models are linear owing to the small-amplitude-displacement hypothesis. For simplicity, the elastic flexure beam model is chosen for the circular cylinder dynamics. The semi-analytical approach used here is based on the methods of Laplace transform in time, *in vacuo* eigenvector expansion with time-dependent coefficients for the transverse beam displacement and separation of variables for the fluid. Moreover, the viscous case is handled with a matched asymptotic expansion performed at first order. The projection of the fluid forces on the *in vacuo* eigenvectors leads to a fully coupled system involving the modal time-dependent displacement coefficients. These coefficients are then obtained by matrix inversion in the Laplace domain and fast numerical inversion of the Laplace transform. The three models, written in the form of convolution products, are described through the analysis of their kernels, involving both the wave propagation phenomena in the fluid domain and the beam elasticity. Last, the three models are illustrated for a specific imposed motion mimicking shock loading. It is shown that their combination permits coverage of a broad range of motions.

Keywords: Semi-analytical method; Asymptotic expansion; Laplace transform; Eigenvector expansion; Transient dynamics; Response function; Shock loading

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Nomenclature

C_i^n	*n*th fluid forces response to an impulsional motion of the whole system
C_m^n	*n*th fluid forces response to an impulsional motion of the *m*th beam mode
c_f	speed of sound in the fluid at rest
E	elastic modulus
F	fluid loading in the $\mathbf{e}_\mathbf{x}$-direction
f_n	*n*th modal time-dependent fluid force coefficient
H_n	beam response function related to the *n*th modal displacement coefficient
I	area moment of inertia
K	Keulegan–Carpenter number
L	beam length
$\mathscr{M}$	density ratio
$\mathscr{M}_a$	Mach number
p	fluid pressure
R_1	circular beam radius
R_2	rigid outer circular cylinder radius
r	radial coordinate
S	beam cross section
s	Laplace variable
t	time
$\mathbf{u}$	fluid velocity field
$\mathscr{W}$	dimensionless frequency
x	relative beam axis displacement in the $\mathbf{e}_\mathbf{x}$-direction
x_i	imposed motion on the whole system
x_n	*n*th time-dependent modal displacement coefficient
z	vertical coordinate
α	confinement ratio
β	Stokes number
γ	aspect ratio
ε	perturbation parameter
λ_n	*n*th *in vacuo* eigenvalue of the beam axis displacement
ν	kinematic fluid viscosity
ρ	dynamic fluid density
ρ_s	beam density
ρ_f	fluid density at rest
ϕ	velocity potential
φ_m^n	projection of the *n*th beam eigenfunction on the *m*th vertical fluid mode
Ω	compressibility number
ω_n	*n*th modal frequency

1. Introduction

When subjected to the effects of underwater explosions, naval propulsion devices can experience highly accelerated high-frequency motions (Keil, 1961; O'Hara and Cunnif, 1993). These components can be in contact with a fluid, as is the case for heat exchangers and nuclear propulsion reactors (Sigrist et al., 2006a, b; Sigrist and Broc, 2007). In order to improve design margins and ensure safety and satisfactory operating performance of the shock-loaded components, precise knowledge of their transient response is required. Since the body motion creates a fluid flow that in turn influences the body motion, complex fluid–structure interaction problems must be taken into account. In a first modelling step, design engineers must identify the meaningful physical phenomena in their particular geometry, such as viscous damping or compressibility effects. This can be achieved by studying simple representative fluid–structure problems for which analytical or semi-analytical solutions are available. The case of a circular cylinder confined in cylindrical fluid domains has already attracted a lot of attention due to its recurrence in the design of naval and nuclear components. However, the numerous studies available in (Gibert, 1986; Chen, 1987) focus on the harmonic dynamics. The transient case, which is of major importance in shock loading, has received comparatively less attention. Although the solution of a time-dependent problem can be formally obtained from the solution of the corresponding harmonic problem using Fourier synthesis methods (Landau and Lifshitz, 1959) or series of resonance modes (Habault and Filippi, 2003, 2004), time-domain methods are believed to provide more physical insight. In addition, these methods are currently receiving renewed interest (Stepanishen, 1997; Iakovlev, 2002, 2004, 2006, 2007), in particular due to increasing computer capabilities. The goal of the present paper is to formulate a time-domain method for a clamped-free elastic circular cylinder, confined in a cylindrical fluid domain initially at rest and subjected to a transient motion along a radial line; see Fig. 1. In addition to its industrial applications, this simple system is of academic concern since it illustrates in a closed form numerous fluid–structure interaction phenomena, such as structural mode coupling by the fluid, viscous damping, and some elasticity–compressibility interaction effects. The most restricting aspect of this study lies in the small-amplitude motion hypothesis. This is nevertheless reasonable for the design of numerous structures and helps lay the foundations for the understanding of more general fluid–structure interaction problems.

In order to cover a broad loading range, three fluid models are considered: potential, viscous and acoustic. Although their corresponding equations are classic (Lamb, 1932), their links and ranges of validity are not always explicit when used separately. In order to get some insight on their *a priori* limits, they are derived here from the compressible Navier–Stokes equations by a formal perturbation method. Owing to the small-amplitude-displacement hypothesis, the resulting fluid models are linear. The fluid forces, expressed for arbitrary motions of their boundaries and put into the