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Channel capacity of OAM based FSO communication systems with partially coherent Bessel–Gaussian beams in anisotropic turbulence

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ABSTRACT

Based on the Rytov approximation theory, the transmission model of an orbital angular momentum (OAM)-carrying partially coherent Bessel–Gaussian (BG) beams propagating in weak anisotropic turbulence is established. The corresponding analytical expression of channel capacity is presented. Influences of anisotropic turbulence parameters and beam parameters on channel capacity of OAM-based free-space optical (FSO) communication systems are discussed in detail. The results indicate channel capacity increases with increasing of almost all of the parameters except for transmission distance. Raising the values of some parameters such as wavelength, propagation altitude and non-Kolmogorov power spectrum index, would markedly improve the channel capacity. In addition, we evaluate the channel capacity of Laguerre–Gaussian (LG) beams and partially coherent BG beams in anisotropic turbulence. It indicates that partially coherent BG beams are better light sources candidates for mitigating the influences of anisotropic turbulence on channel capacity of OAM-based FSO communication systems.

1. Introduction

The vortex beam defines an infinite dimensional Hilbert space; theoretically, each photon can carry unlimited OAM states [1,2]. Meanwhile, because the OAM spatial domain is independent of other physical dimensions of light wave, OAM multiplexing can combine with other existing multiplexing techniques [3,4], which has the potential to extremely improve the capacity of communication systems. However, in practical applications, atmospheric turbulence obviously attenuate the performance of OAM-based FSO communication systems, such as, reducing channel capacity, crosstalk among channels and inducing the spread of spiral spectrum of OAM modes, etc. [5–7]. Especially, when the atmosphere turbulence gets relatively strong, in order to maintain the performance of the communication systems, it is necessary to adopt turbulence compensation techniques, such as signal processing-based (electrical domain) and adaptive optics-based (optical domain) techniques [8–10]. Considering improving the channel capacity, it is significant to reveal the influence factors of channel capacity of OAM-based FSO communication systems. So far, theoretical expressions of channel capacity of OAM-based FSO communication links with LG beams in atmospheric turbulence have been studied for Kolmogorov spectrum [5,11], isotropic non-Kolmogorov spectrum [12], and modified Von Karman spectrum [13]. Most of the communication systems mentioned above are focused on the transmission of classical LG beams in isotropic atmospheric turbulence. However, experimental and theoretical results [14–16] show that the atmospheric turbulence is inhomogeneous and anisotropic, especially in the stable layered stratosphere. Thus, the common isotropic Kolmogorov spectrum and non-Kolmogorov spectrum may not adequately describe the real behavior of atmospheric turbulence. In 2015, an anisotropic non-Kolmogorov spectrum model was proposed [17], which took into account not only the asymmetric property of turbulence eddies in horizontal and vertical directions, but also the finite turbulence outer and inner scales effects. In addition, compared to the LG beams, non-diffractive vortex beams can effectively weaken the effects of atmospheric turbulence [18–20]. Besides, it shows that atmospheric turbulence has less effect on partially coherent beams than fully coherent beams [21–23]. Therefore, partially coherent BG beams are good light sources for reducing the influences of atmospheric turbulence on the transmission of OAM modes due to their non-diffraction and self-healing properties. To the best of our knowledge, not much attention has been paid to anisotropic turbulence influences on channel capacity of the OAM modes for partially coherent BG beams.

In this paper, we mainly study the channel capacity of partially coherent BG beams propagating in weak anisotropic non-Kolmogorov

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turbulence. We research the effects of asymmetric anisotropic turbulence eddies and beam parameters on the channel capacity. We simulate the dependences of channel capacity on different parameters in detail. Our work would provide theoretic reference for practical OAM-based FSO communication between airplanes at high altitude.

2. Analysis of theory

In free space region with the propagation distance z, the complex amplitude of BG beams can be represented as [24]

$$u_0(\rho,\phi,z)=q^{-1}\exp\left(ikz-\frac{\rho^2}{q{w_0}^2}+\frac{\eta^2}{4q}-\frac{\eta^2}{4}\right)J_{m_0}\left(\frac{\eta\rho}{qw_0}\right)\exp\left(im_0\phi\right) \quad (1)$$

where $q=1+iz/z_0$, $z_0=kw_0^2/2$, z_0 is the Rayleigh range; $\rho=(\rho,\phi)$ is the 2-D position vector in the source plane; $J_n()$ is the nth order Bessel function of the first kind; m_0 is the topological charge of OAM carried by BG beams; η is a constant determining the beam profile; w_0 is the beam waist; φ is the azimuthal angle; λ is the wavelength; $k=2\pi/\lambda$ is the wave number.

Using the Rytov approximation [25], the complex amplitude of partially coherent BG beams in a weak turbulence fluctuation region can be given as [18,26]

$$u(\rho,\phi,z)=u_0(\rho,\phi,z)\exp\left[i\varphi_S(\rho)+\psi_1(\rho,z)\right] \quad (2)$$

where $\psi_1(\rho,z)$ denotes the complex phase perturbation of spherical waves propagating in anisotropic turbulence; $\varphi_s(\rho)$ is the random phase perturbation caused by the phase diffuser.

According to the discussion in [27], the function $u(\rho,\phi,z)$ can be written as a superposition of the spiral harmonics $\exp(im\phi)$

$$u(\rho,\phi,z)=\frac{1}{\sqrt{2\pi}}\sum_{m=-\infty}^{\infty}\beta\left(m|m_0\right)\exp\left(im\phi\right) \quad (3)$$

where $\beta\left(m|m_0\right)$ is given by the integral

$$\beta\left(m|m_0\right)=\frac{1}{\sqrt{2\pi}}\int_0^{2\pi}u(\rho,\phi,z)\exp(-im\phi)d_\phi \quad (4)$$

Following the same process as in [18], the ensemble average of mode probability distribution for partially coherent BG beams in the paraxial channel can be described as

$$\left\langle\left|\beta\left(m|m_0\right)\right|^2\right\rangle=\frac{1}{2\pi}\iint u_0(\rho,\phi,z)\,u_0^*\left(\rho',\phi',z\right)\times\exp\left[-im(\phi-\phi')-(\left|\rho-\rho'\right|^2/\widetilde{\rho}_0^2)\right]d_\phi d_{\phi'} \quad (5)$$

where $\langle\rangle$ denotes an ensemble average over the turbulence statistics. $\widetilde{\rho}_0^2=\left(1+\rho_0^2/\rho_s^2\right)^{-1/2}\times\rho_0$, where ρ_s is the source spatial coherence length, which depicts the partial coherence properties of the beam source. $\widetilde{\rho_0}$ and ρ_0 (the expression is described in next paragraph) denote the effective spatial coherence length and spatial coherence length of a spherical wave propagating in anisotropic non-Kolmogorov turbulence, respectively. The effective spatial coherence length $(\widetilde{\rho_0})$ considers the influences of both the coherence length of phase fluctuations in atmospheric turbulence (ρ_0) and the spatial coherence of beam sources (ρ_s).

ρ_0 can be expressed as [17,18]

$$\rho_0=\left\{M\left(\xi_x,\xi_y\right)\frac{\pi^2k^2zA(\alpha)\widetilde{C_n^2}}{6(\alpha-2)}\times\left[k_m^{2-\alpha}\gamma\exp\left(\frac{k_0^2}{k_m^2}\right)\Gamma\left(2-\frac{\alpha}{2},\frac{k_0^2}{k_m^2}\right)-2k_0^{4-\alpha}\right]\right\}^{-1/2} \quad (6)$$

where $\Gamma(x,y)$ is the incomplete Gamma function, α is the non-Kolmogorov power spectrum index with $3<\alpha<4$, l_0 and L_0 represent the inner scale and outer scale, respectively. $A(\alpha)=\Gamma(\alpha-1)\cos(\pi\alpha/2)/(4\pi^2)$, $\gamma=2k_0^2-2k_m^2+\alpha k_m^2$, $k_0=2\pi/L_0$, $k_m=c(\alpha)/l_0$, $c(\alpha)=\{A(\alpha)\,\Gamma[(5-\alpha)/2]2\pi/3\}^{1/(\alpha-5)}$, $M(\xi_x,\xi_y)=(\xi_x^2+\xi_y^2)/2\xi_x^2\xi_y^2$, $M(\xi_x,\xi_y)$ denotes the general anisotropic factor, ξ_x and ξ_y represent anisotropy parameters along the x and y directions, respectively. When $\xi_x=\xi_y=1$, anisotropic non-Kolmogorov spectral model becomes a conventional isotropic non-Kolmogorov spectral model. $\widetilde{C_n^2}=\beta C_n^2$, $\widetilde{C_n^2}$ is a generalized structure parameter and the unit is $m^{3-\alpha}$, β is a dimensional constant and the unit is $m^{-\alpha+11/3}$, $C_n^2=5.94\times10^{-53}(v/27)^2h^{10}\exp(-h/1000)+2.7\times10^{-16}\exp(-h/1500)+C_n^2(0)\exp(-h/100)$, h is the altitude and v is wind speed in high altitude, $C_n^2(0)$ is the refractive index structure constant on the ground.

Substituting (1) into (5) and utilizing the integral expression [28]

$$\int_0^{2\pi}\exp\left[-in\varphi_1+\eta\cos\left(\varphi_1-\varphi_2\right)\right]d\varphi_1=2\pi\exp\left(-in\varphi_2\right)I_n(\eta) \quad (7)$$

where $I_n(\eta)$ is the first kind of modified Bessel function with n order.

Thus, the final expression of $\left\langle\left|\beta\left(m|m_0\right)\right|^2\right\rangle$ can be expressed as:

$$\left\langle\left|\beta\left(m|m_0\right)\right|^2\right\rangle=\left(1+\frac{z^2}{z_0^2}\right)^{-1}\exp\left(-\frac{2\rho^2}{\widetilde{\rho}_0^2}\right)\exp\left[-\left(\frac{2\rho^2}{w_0^2}+\frac{\eta^2}{2}\right)\left(1+\frac{z^2}{z_0^2}\right)^{-1}-\frac{\eta^2}{2}\right]\times\left|J_{m_0}\left(\frac{\eta\rho}{qw_0}\right)\right|^2 I_{m_0-m}\left(\frac{2\rho^2}{\widetilde{\rho}_0^2}\right) \quad (8)$$

We can calculate OAM mode detection probability $P(m|m_0)$ (i.e. $m=m_0$) and crosstalk probability $P(m|m_0)$ (i.e. $m=m_0\pm\Delta m$, $\Delta m\neq0$) of partially coherent BG beams propagation in anisotropic turbulence by the following relationship [19,27]:

$$P\left(m|m_0\right)=\frac{\int_{-\infty}^{\infty}\left\langle\left|\beta\left(m|m_0\right)\right|^2\right\rangle\rho d_\rho}{\sum_{l=-\infty}^{\infty}\int_{-\infty}^{\infty}\left\langle\left|\beta\left(l|m_0\right)\right|^2\right\rangle\rho d_\rho} \quad (9)$$

where m_0 is signal OAM mode; m and l are arbitrary OAM mode of partially coherent BG beams; Δm represents the OAM quantum number difference.

According to the aforementioned formulas, we can quantitatively study the influences of anisotropic non-Kolmogorov turbulence on the channel capacity of OAM-based FSO communication links with partially coherent BG beams. Here, we select signal OAM modes $m_0$in the range of $l=-L,\ldots L$, which forms an $N=(2L+1)$ dimension Hilbert space.

Based on the literature [5], assuming that FSO link is a discrete memoryless channel, the channel capacity of partially coherent BG beams propagation in anisotropic turbulence can be calculated as:

$$C=\max\left[H\left(m_0\right)-H\left(m_0|m\right)\right]=\log_2(N)+\frac{1}{N}\sum_{m=-\infty}^{\infty}\sum_{m_0=-L}^{L}P\left(m|m_0\right)\left[\log_2P\left(m|m_0\right)-\log_2\sum_{m_0=-L}^{L}P\left(m|m_0\right)\right] \quad (10)$$

where $H(m_0)$is the entropy of the source, $H(m_0|m)$ is the conditional entropy.

3. Numerical simulations and discussions

In this section, we investigate the channel capacity of the OAM based FSO communication links with partially coherent BG beams propagating in weak anisotropic turbulence by using aforementioned analytical equations. The numerical results have been discussed in detail.

Fig. 1 depicts the impact of transmission distance and the number of transmitted OAM modes on channel capacity. The simulation parameters are set as follows: beam shape parameter $\eta=10$, beam width $w_0=0.1$ m, $\beta=1$, non-Kolmogorov power spectrum index $\alpha=3.37$, wavelength $\lambda=1550$ nm, source coherence length $\rho_s=1$ m, wind speed $v=21$ m/s, turbulence inner scale $l_0=1$ mm, turbulence outer scale $L_0=50$ m, propagation altitude $h=3$ km, anisotropy coefficients