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Optimal reference polarization states for the calibration of general Stokes polarimeters in the presence of noise

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ABSTRACT

During the calibration of the system matrix of a Stokes polarimeter using reference polarization states (RPSs) and pseudo-inversion estimation method, the measurement intensities are usually noised by the signal-independent additive Gaussian noise or signal-dependent Poisson shot noise, the precision of the estimated system matrix is degraded. In this paper, we present a paradigm for selecting RPSs to improve the precision of the estimated system matrix in the presence of both types of noise. The analytical solution of the precision of the system matrix estimated with the RPSs are derived. Experimental measurements from a general Stokes polarimeter show that accurate system matrix is estimated with the optimal RPSs, which are generated using two rotating quarter-wave plates. The advantage of using optimal RPSs is a reduction in measurement time with high calibration precision.

1. Introduction

Stokes polarimeters, also termed as polarization state analyzers (PSAs), are powerful tools for characterizing the states of polarization of target [1–3]. To get full Stokes parameters [S_0, S_1, S_2, S_3], a PSA should have $N_B \geq 4$ different analysis states, and these states form a so-called system matrix B with a dimension of $N_B \times 4$ [1]. Before using the PSA for practical measurement, its practical system matrix B needs to be estimated using a polarization state generator (PSG) that generates $N_A \geq 4$ different reference polarization states (RPSs) [4,5]. These states form a RPS matrix A with a dimension of $N_A \times 4$. The system matrix B is estimated using pseudo-inversion estimation of the well-known RPS matrix A and the measured intensity matrix I with a dimension of $N_A \times N_B$. The RPS method for calibration has the potential of accounting for higher order effects of systematic errors such as multiple reflections between or within optical devices, incorrectly oriented crystals in retarders, imperfect polarizers, and residual birefringence [4,5]. However, there are no guide theory for the selection of RPSs. Most of RPSs are generated using a simple PSG setup of easy implementation [6–10].

It is noted that the intensities, measured by the PSA during calibration, usually are perturbed by several types of noise such as signal-independent detector noise, signal-dependent shot noise, or compound noise [11]. The estimation precision of the system matrix is then limited by noisy data. Up to now, most researches only focus on the optimization of the PSA's analysis states in the presence of noise [12–21]. For example, the optimal estimation of samples' Muller matrix by selecting PSG and PSA in the presence of both Gaussian and Poisson noise are presented [21]. The closed-form solutions of estimation precision show that the optimal PSG and PSA architectures that minimize and equalize the estimation variances of the sample's Muller matrix are based on spherical designs of order 2 or 3. These spherical designs have ever been identified analytically [14–16] and numerically [17–19] during the optimization of the PSA [10–12]. A spherical design of order t is a set of N_A points on the surface of the unit sphere for which the normalized integral of any polynomial of degree t or less is equal to the average taken over the N_A points [14]. The platonic solids such as tetrahedron, octahedron, cube, icosahedron, dodecahedron all belong to spherical designs [16,17].

However, to our best knowledge, the choice of RPSs for minimizing and equalizing the estimation variance of the system matrix of a general PSA has not been explored, and the corresponding estimation performance remains to be quantified. In this paper, we will estimate the practical system matrix of a general PSA in linear optics with pseudo-inversion method. The closed-form expressions of the estimation precision in the presence of both Gaussian and Poisson noise are derived. It is demonstrated that, for calibrating arbitrary system matrix (regardless of optimal ones [12–19], or not [8,22–27]), the optimal RPSs that can minimize and equalize noise variance are based on spherical designs of order 2 or 3. The important feature of the optimal RPSs is

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that the sum of the matrix elements of any columns is equal to zero except for the first column. We verify the analytical solutions with Monte Carlo simulations and experiments at first time. The architectures for generating the optimal RPSs are presented. Experimental results show that the system matrix estimated with the proposed RPSs has immunity to kinds of noise such as, but not limit to, Gaussian and Poisson noise.

2. Theory

2.1. Calibration model

The calibration system usually comprises an unpolarized light source with intensity I_0, a PSG and a PSA. The RPSs and analysis states generated by the PSG and PSA, respectively, are stacked row-wise to form the RPS matrix A and the system matrix B. The matrix A (B) thus has the dimensions $N_A \times 4$ ($N_B \times 4$). The intensities acquired by the PSA are

$$I = I_0 B A^T, \tag{1}$$

where I is a $N_B \times N_A$-dimensional intensity matrix representing $N_B N_A$ measurements, and the superscript T indicates transpose. In the following, we will estimate the system matrix $I_0 B$, and define a vector operator by stacking the matrix elements row-wise to form a vector. Then Eq. (1) is rewritten as

$$\mathbf{V}_I = [E \otimes A]\mathbf{V}_B, \tag{2}$$

where $\otimes$ represents the Kronecker product [28], E is a $N_B\times N_B$-dimensional identity matrix, $\mathbf{V}_B = ([V_B]_1, \ldots, [V_B]_{4N_B})$ is a $4N_B$-dimensional analysis state vector, and $\mathbf{V}_I = ([V_I]_1, \ldots, [V_I]_{N_A N_B})$ is a $N_A N_B$-dimensional intensity vector by reading the corresponding matrix elements in the lexicographic order.

In this paper, we consider that the measurement vector $\mathbf{V}_I$ is disturbed by two types of common noise sources: additive Gaussian noise or Poisson shot noise, respectively. The analysis state vector $\mathbf{V}_B$ is estimated from the noisy measurements using pseudo-inverse (PI) estimator $\widehat{\mathbf{V}}_B$ [21],

$$\widehat{\mathbf{V}}_B = P\mathbf{V}_I \text{ with } P = \left([E \otimes A]^T [E \otimes A]\right)^{-1} [E \otimes A]^T, \tag{3}$$

where P is the pseudoinverse of the $4N_B\times N_A N_B$-dimensional matrix $[E \otimes A]$. Based the properties of the Kronecker product [26], the PI matrix is rewritten as

$$P = [G_E \otimes G_A][E \otimes A]^T, \tag{4}$$

where $G_U = (U^T U)^{-1}$ with $U = E$ or A.

The pseudo-inverse estimator $\widehat{\mathbf{V}}_B$ is the best possible estimator and unbiased in the presence of Gaussian or Poisson noise. Its precision is indicated by its covariance matrix [21].

$$\Gamma_{\mathbf{V}_B} = P\Gamma_{\mathbf{V}_I} P^T. \tag{5}$$

A standard scalar performance criterion for polarization calibration is the sum of the variances of all the elements of the system matrix, which is the trace of $\Gamma_{\mathbf{V}_B}$ [21]:

$$\Omega = \mathrm{Tr}[P\Gamma_{\mathbf{V}_I} P^T]. \tag{6}$$

2.2. Gaussian noise

We first assume that the measurements are mainly perturbed by zero-mean additive white Gaussian noise with variance σ^2. The covariance matrix in Eq. (5) of the estimator should be [21]

$$\Gamma_{\mathbf{V}_B} = \sigma^2 [G_E \otimes G_A], \tag{7}$$

and the criterion in Eq. (6) is deduced as

$$\Omega^{\mathrm{gau}} = \sigma^2 \mathrm{Tr}[G_E]\mathrm{Tr}[G_A], \tag{8}$$

where $\mathrm{Tr}[G_E] = \mathrm{Tr}[E] = N_B$ is a constant. Obviously, the total variance does not depend on the observed system matrix itself. Our aim is to find optimal RPS matrix A for minimizing the performance criterion Ω^{gau}, thus minimizing $\mathrm{Tr}[G_A]$. It has been shown that $\mathrm{Tr}[G_A]$ is minimized if the last three columns of the RPS matrix A form a sphere 2 design on the Poincaré sphere of unit radius [14–16], that is $A^T A = \frac{N_A}{12}\mathrm{diag}(3, 1, 1, 1)$ and $G_A = \frac{4}{N_A}\mathrm{diag}(1, 3, 3, 3)$. Then the matrix $G_E \otimes G_A$ in Eq. (7) is a $4N_B \times 4N_B$-dimensional diagonal matrix, and its coefficients $[G_E \otimes G_A]_{ii}$ are derived as

$$\begin{cases} 4/N_A & \text{if } i = 4m + 1, \text{ and } m = 0, 1, \ldots, N_B - 1 \\ 12/N_A & \text{others.} \end{cases} \tag{9}$$

The corresponding minimal value of the performance criterion is calculated as

$$\Omega^{\mathrm{gau}}_{\mathrm{opt}} = \frac{40N_B}{N_A}\sigma^2. \tag{10}$$

As seen, the total noise variance decreases with the increase of the number of the RPSs when the numbers of the analysis states are fixed. The covariance matrix $\Gamma_{\mathbf{V}_B}$ in Eq. (7) is diagonal and its diagonal elements denote the estimation variances of the elements of the system matrix B as

$$\mathrm{VAR}[B]^{\mathrm{gau}}_{\mathrm{opt}} = \frac{4}{N_A}\sigma^2 \begin{bmatrix} 1 & 3 & 3 & 3 \\ 1 & 3 & 3 & 3 \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 3 & 3 & 3 \end{bmatrix}_{N_B\times 4}. \tag{11}$$

It is interesting to note that the variance of each element is also independent of the observed system matrix B, and its last three columns achieve noise equalization.

2.3. Poisson noise

Second, we assume that the measurements are mainly degraded by Poisson shot noise. The diagonal element of the covariance matrix $\Gamma_{\mathbf{V}_B}$ in Eq. (5) is derived as [21]

$$\forall i \in [1, 4N_B], [\Gamma_{\mathbf{V}_B}]_{ii} = \sum_{j=1}^{4N_B} Q_{ij}[\mathbf{V}_B]_j, \tag{12}$$

where Q is a $4N_B \times 4N_B$-dimensional matrix expressed as

$$\forall (i, j) \in [1, 4N_B],\ Q_{ij} = \sum_{n=1}^{N_A N_B} (P_{in})^2 [E \otimes A]_{nj}. \tag{13}$$

It is easily found that the variances depend on the observed system matrix B in the presence of Poisson noise, which are different from the case in Eq. (7) in the presence of Gaussian noise. The performance criterion in Eq. (6) is then given as [21]

$$\Omega^{\mathrm{poi}} = \sum_{i=1}^{4N_B} [\Gamma_{\mathbf{V}_B}]_{ii} = \mathbf{V}^T_{(E,A)}\mathbf{V}_B, \tag{14}$$

where $\mathbf{V}_{(E,A)}$ is a $4N_B$-dimensional vector defined as

$$\forall j \in [1, 4N_B], [\mathbf{V}_{(E,A)}]_j = \sum_{i=1}^{4N_B} Q_{ij}. \tag{15}$$

If the RPSs form a sphere 3 design on the Poincaré sphere [21], one derives

$$Q_{ij} = \begin{cases} 1/N_A & \text{if } i = m \cdot 4 + 1, \text{ and } m = 0, 1, \ldots, N_B - 1 \\ 3/N_A & \text{others.} \end{cases} \tag{16}$$

Then the optimal value of the performance criterion is derived as

$$\Omega^{\mathrm{poi}}_{\mathrm{opt}} = \frac{40N_B}{N_A}\frac{I_0}{4}. \tag{17}$$