Optics Communications 416 (2018) 181–184

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Goos–Hanchen shifts in tilted uniaxial crystals

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ARTICLE INFO

Keywords:
Goos–Hanchen shift
Tilted uniaxial crystal
Permittivity

ABSTRACT

The Goos–Hanchen shifts at the surface of the tilted uniaxial crystals have been studied with the help of the stationary phase method. It is found that the permittivity and the optical axis of the uniaxial crystal have outstanding influence on the Goos–Hanchen shift. The numerical results show that the negative Goos–Hanchen shift can occur even when the refractive index of the material is not negative. Besides, the Goos–Hanchen shift can be negative or positive infinite under certain conditions. Our results may provide useful information in manipulating the Goos–Hanchen shift in uniaxial crystals. We believe this method could find practical applications in tunable sensors and switches, which are based on Goos–Hanchen shifts.

1. Introduction

The displacement of a light beam upon total internal reflection at the boundary of two different materials is known as Goos–Hanchen (GH) shift, which was first observed by Goos and Hanchen [1]. During the past decades, the GH shifts of various materials have been extensively investigated, regardless of isotropic [2–7] or anisotropic materials [8–13]. Taking anisotropic materials for examples, Huang et al. recently studied the GH shift in a uniaxial anisotropic chiral metamaterial slab [11]. Chern investigated the effect of damping on the GH shift from weakly absorbing anisotropic metamaterials [12]. Although there were many studies on the GH shifts in anisotropic materials, most of the work have been done by considering the optical axis (OA) is perpendicular or parallel to the interface. To the author's knowledge, few interests has been paid when the OA of the anisotropic material is tilted off. Wang et al. have studies the influence of the orientation of the OA on the GH shift, but their discussion was limited to left-handed materials [13]. In fact, other types of permittivity will lead to different results because of the complex dispersion of the electromagnetic waves in the material. In this paper, we study the impact of the orientation of the OA and the permittivity of the uniaxial crystal on the GH shift. It is found that the negative GH shift can occur when the material is not left-handed material. Besides, the numerical results reveal that the GH shift can be flexibly tuned by adjusting the optical axis of the uniaxial crystal. This may enable us to fulfill tunable GH shift based optical devices such as sensors and switches, since the ability to tune the shift values is desirable for these applications [14].

2. Formulation

The configuration of the uniaxial crystal with tilted OA is shown in Fig. 1, which the OA is tilted off the z-axis with an angle of ϕ. The permittivity tensor of the uniaxial crystal can be expressed as $\boldsymbol{\varepsilon} = \mathrm{diag}\left(\varepsilon_\perp, \varepsilon_\perp, \varepsilon_\parallel\right)$ when its OA is along the z-axis, where $\varepsilon_\perp$ and $\varepsilon_\parallel$ indicate the two components perpendicular and parallel to the OA. When its OA is tilted with an angle of ϕ, the permittivity tensor in the xyz coordinate system can be expressed as [15,16]

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_\perp \cos^2\phi + \varepsilon_\parallel \sin^2\phi & 0 & \left(\varepsilon_\parallel - \varepsilon_\perp\right)\sin\phi\cos\phi \\ 0 & \varepsilon_\perp & 0 \\ \left(\varepsilon_\parallel - \varepsilon_\perp\right)\sin\phi\cos\phi & 0 & \varepsilon_\parallel \cos^2\phi + \varepsilon_\perp \sin^2\phi \end{pmatrix}. \tag{1}$$

Here we consider the incidence wave is a transverse magnetic (TM) wave with the x–z plane as the plane of incidence. In this case, there is no polarization coupling in the medium and for a transverse electric (TE) wave, the medium behaves like an isotropic medium with a permittivity $\varepsilon_\perp$, regardless of the tilting. The GH shifts of isotropic materials have been studies in many published literatures [2–7], therefore we limit our discussion on TM wave only in this paper. The impact of the anisotropy including the absorption on the GH shift is quite complicated, so we take $\varepsilon_\perp$ and $\varepsilon_\parallel$ as real numbers in the calculation.

The dispersion of the electromagnetic waves with angular frequency ω propagating in the tilted uniaxial crystal can be described by [15]

$$\frac{\left(k_z \cos\phi + k_x \sin\phi\right)^2}{\varepsilon_\perp} + \frac{\left(k_z \sin\phi - k_x \cos\phi\right)^2}{\varepsilon_\parallel} = k_0^2 \tag{2}$$

where k_x and k_z represent the x and z components of the wavevector. $k_x = k_0 \sin\theta$, where θ is the angle of incidence and $k_0 = \omega/c_0$ with c_0 being the speed of light in the vacuum. The z-components of the forward and backward wavevector are given by [15]

$$k_z^\pm = \frac{-k_x\left(\varepsilon_\parallel - \varepsilon_\perp\right)\sin\phi\cos\phi \pm k_0\sqrt{\varepsilon_\perp\varepsilon_\parallel\left(\varepsilon_\parallel \cos^2\phi + \varepsilon_\perp \sin^2\phi - \sin^2\theta\right)}}{\varepsilon_\parallel \cos^2\phi + \varepsilon_\perp \sin^2\phi}. \tag{3}$$

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https://doi.org/10.1016/j.optcom.2018.02.020
Received 10 January 2018; Received in revised form 3 February 2018; Accepted 8 February 2018

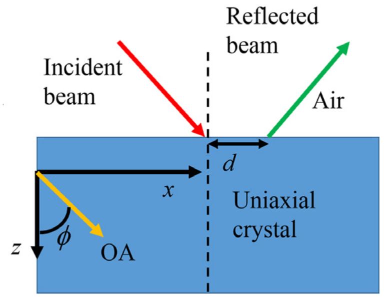

Fig. 1. Schematic diagram of the GH shift. The OA of the uniaxial crystal is tilted off the z axis with an angle of ϕ.

When the incident beam is totally reflected, there will be

$$\varepsilon_\perp\varepsilon_\parallel\left(\varepsilon_\parallel\cos^2\phi+\varepsilon_\perp\sin^2\phi-\sin^2\theta\right)<0. \tag{4}$$

The reflection coefficient in the interface is [15]

$$r=\frac{\sqrt{\varepsilon_\perp\varepsilon_\parallel\cos^2\theta}-j\sqrt{\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi}}{\sqrt{\varepsilon_\perp\varepsilon_\parallel\cos^2\theta}+j\sqrt{\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi}}. \tag{5}$$

We have to address that which root in Eq. (3) is in the forward direction does not matter, because it does not have impact on the reflection coefficient. Besides, the method to determine which root is for the forward direction can be found in Ref. [16]. Then the phase of the reflected beam can be expressed as

$$\varphi=-2\arctan\left[\sqrt{\frac{\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi}{\varepsilon_\perp\varepsilon_\parallel\cos^2\theta}}\right]. \tag{6}$$

According to the stationary phase method, the GH shift of the reflected beam can be calculated by [5]

$$d=-\frac{\lambda}{2\pi}\frac{d\varphi}{d\theta} \tag{7}$$

where λ is the wavelength of the incidence beam. Submitting Eq. (6) into Eq. (7), we can get

$$\frac{d}{\lambda}=\frac{1}{\pi}\frac{\left(1-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi\right)\sin\theta}{\varepsilon_\perp\varepsilon_\parallel\cos^2\theta+\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi}\times\sqrt{\frac{\varepsilon_\perp\varepsilon_\parallel}{\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi}}. \tag{8}$$

With the help of Eq. (8), one can investigate the influence of the OA and the permittivity on the GH shift.

3. Results and discussions

We first consider the simpler case when the OA is in the z-axis. The d/λ as functions of the two permittivity components are calculated when the angles of incidence are 30° and 60°, respectively, as shown in Fig. 2. In the permittivity space, we can find six different subspaces, which the GH shift presents different performance.

(1) When $\varepsilon_\perp<0, \varepsilon_\parallel<0$, the beam is totally reflected and Eq. (4) can be always satisfied regardless of the angle of incidence. Thus the GH shift is not zero in this region.
(2) When $\varepsilon_\perp<0, \varepsilon_\parallel>1$, this case is similar to case (1). The GH shift is positive in these two cases.
(3) When $\varepsilon_\perp<0, 0<\varepsilon_\parallel<1$, whether the GH shift is zero or not is determined by the angle of incidence. The GH shift is negative in this case when there is GH shift. This information can also be obtained by analyzing Eq. (8).
(4) When $\varepsilon_\perp>0, \varepsilon_\parallel<0$, Eq. (4) cannot be satisfied, thus the GH shift is always zero.
(5) When $\varepsilon_\perp>0, \varepsilon_\parallel>1$, Eq. (4) cannot be satisfied, thus there is no GH shift. This case is similar to case (4).
(6) When $\varepsilon_\perp>0, 0<\varepsilon_\parallel<1$, this case is similar to case (3), where the GH shift is determined by the angle of incidence. However, the different is that the GH shift is always positive.

Based on above analysis, we can see that the different values of the two permittivity components have great impact on the GH shift. The GH shift can be negative even when the material is not the left-handed materials. Next we consider the GH shift when the OA is not perpendicular to the interface.

Fig. 3 shows the results when the angle of incidence is 45° the tilted angles are 30°, 60° and 90°, respectively. Taking Fig. 3(a) for example, there is a clear dividing line in the permittivity space. The expression of the line is

$$\sin^2\theta-\varepsilon_\parallel\cos^2\phi-\varepsilon_\perp\sin^2\phi=0. \tag{9}$$

The GH shift tends to be infinite in this line, except at two points when $\varepsilon_\perp$ or $\varepsilon_\parallel$ is zero. The GH shift tends to be negative infinite when the two permittivity components satisfy $\varepsilon_\perp\varepsilon_\parallel<0$, whereas the GH shift tends to be positive infinite when the two permittivity components satisfy $\varepsilon_\perp>0, \varepsilon_\parallel>0$. The similar results can also be seen in Fig. 3(b). When the tilted angle is 0° and 90°, as shown in Figs. 2 and 3(c), the dividing line is vertical and horizontal in the space, respectively. We have to address that the GH shift tends to be infinite in the dividing line, but the range of the colorbar is set from −1 to +1. It is for better exhibition, otherwise the GH shift will not be obvious in other areas, except in the dividing line.

Besides, one can see that in Fig. 3(a), there will be GH shift in the region where $\varepsilon_\perp<0$, $\varepsilon_\parallel>0$, and there will be no GH shift in the region where $\varepsilon_\perp>0$, $\varepsilon_\parallel<0$. However, the results are opposite in Fig. 3(b). The results in these two pictures are calculated for different tilted angles. Therefore, the different orientation of OA can influence the GH shift a lot. This character is a good advantage for uniaxial crystal to realize tunable optical devices, which are based on GH shift, such as sensors and switches. Compared with another way to achieve tunable GH shift, which is tuned by dynamically adjusting the Fermi level of the graphene [14], the method presented in this paper is very different. Besides, some attention has been paid to the perfect absorption of the metamaterials whose optical axis are tilted with respect to the surface normal [16,17]. With the development of the metamaterials, we believe more and more attention will be focused on the GH shift of metamaterials with tilted optical axis.

4. Conclusion

In summary, we have investigated the function of the OA and the permittivity on the GH shift in the tilted uniaxial crystals. The condition for total internal reflection is obtained and the explicitly expression of the GH shift is derived by using stationary phase method. It is found that the negative GH shift can occur when the two permittivity components have the opposite signs. Besides, the GH shift can tend to be positive infinite when the two permittivity components have the same signs, whereas the GH shift can tend to be negative infinite when the two permittivity components have the opposite signs. Our results show the rich manipulation on the GH shift in uniaxial crystals by tailoring the permittivity and adjusting the OA. This may help us design the optical devices such as tunable sensors and switches, which are based on GH shifts.

Acknowledgment

This work was supported by the China Scholarship Council (No. 201706010271)