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# Coupled-resonator waveguide perfect transport single-photon by interatomic dipole–dipole interaction

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ABSTRACT

We theoretically investigate single-photon coherent transport in a one-dimensional coupled-resonator waveguide coupled to two quantum emitters with dipole–dipole interactions. The numerical simulations demonstrate that the transmission spectrum of the photon depends on the two atoms dipole–dipole interactions and the photon–atom couplings. The dipole–dipole interactions may change the dip positions in the spectra and the coupling strength may broaden the frequency band width in the transmission spectrum. We further demonstrate that the typical transmission spectra split into two dips due to the dipole–dipole interactions. This phenomenon may be used to manufacture new quantum waveguide devices.

## 1. Introduction

Recently, the development of cavity quantum electrodynamics ($QED$) [1–3] has provided a ideal platform for the studies of quantum-information processing [4–9] and the development of impressive architectures and the test of fundamental quantum mechanics. In cavity $QED$, atoms are placed inside of a high-finesse electromagnetic resonator with a discrete radiation spectrum. Within the bounding between the resonator mirrors, a single photon can effectively interact with the atoms many times, which leads to a significant enhancement of the atom-photon coupling [10]. This relatively strong coupling has attracted great interests to the study of the single-photon transport, especially in a low-dimensional coupled-resonator waveguide($CRW$), for its extensive application in quantum-information processing [11] and the realization of photonic devices. The interactions between the emitters induced by the waveguide modes are long range [12], making the low-dimensional system a very simple and excellent platform for studying many-body physics. The $CRW$ which can be realized by photonic crystals [13] or superconductor transmission line resonators is an important physical mode [14]. Until now, in one-dimensional $CRW$, many theoretical [15–20] and experimental [21–27] studies on single-photon transport properties have been reported. Compare to the extremely weak interaction induced by photon–photon scattering, the light-matter coupling in the $CRW$ can lead to a relatively strong interactions among single photons. This makes photons excellent long-distance carriers of classical and quantum information.

Considering radiation spectra of multi-atom systems in a cavity field, previous studies have neglected the interaction between atoms [16,19,28–30]. In fact, when atoms are close together, there should be interactions between the atoms, for example, dipole–dipole interactions ($DDI$) created by the exchange of virtual photons. It is well-known that if the separation between two atoms is much smaller than the resonance wavelength, the $DDI$ can be strengthened. Moreover, the $DDI$ can induce new physical phenomena in the physical system and have many applications in quantum information processing. For instance, Cheng et al. theoretically investigated single photon transport in a one-dimensional waveguide coupled to a nanocavity embedded with two atoms with $DDI$ [31]; Yu et al. studied the effect of interatomic $DDI$ on a single-photon transmission spectrum in a single-mode optical waveguide containing a pair of two-level atoms with dipole interactions and the incident photon [32]; Pellegrino et al. experimentally studied the emergence of collective scattering in the presence of $DDI$ when they illuminated a cold cloud of $^{87}$Rb atoms with a near-resonant and weak

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intensity laser [33]. In this paper, we consider the problem of photon transport in a one-dimensional coupled-resonator waveguide, where two two-level systems with $DDI$ are embedded at the central site. When the emitter separation is much smaller than the resonant wavelength, the emitter dynamics and emission spectra can be significantly modified by $DDI$. Comparing the results with and without $DDI$ to examine the shape changes in the transmission spectra, we observe that the coupling strength can broaden the frequency band width in the transmission spectra and $DDI$ change the dips position in the spectra. In addition, when the coupling strength between the two atoms and the cavity is not equal, the typical transmission spectrum splits into two dips. Our theory may provide an important tool for studying many-body physics and designing new waveguide-based quantum devices.

## 2. Mode and solution

As shown in Fig. 1, our mode consists of a one-dimensional $CRW$ and two-level atoms (Here, we chose $^{87}$Rb atoms.), and the two-level atoms are embedded at the central sites. The atoms play an essential role in controlling the propagation of a single photon and the two-level atoms between the ground state $|g\rangle$ and excited state $|e\rangle$. The Hamiltonian for the $CRW$ is given by

$$H_c = \sum_j [\omega_c a_j^\dagger a_j - \zeta(a_j^\dagger a_{j+1} + h\cdot c)] \tag{1}$$

with the intercavity coupling constant $\zeta$, describes the photon hopping from one cavity to another. We assume that all the resonators have the same frequency $\omega_c$ and $\hbar = 1$. Under the rotating-wave approximation, the Hamiltonian of the atoms and the Hamiltonian of the two-level atoms interacting with the cavity fields are

$$H_a = \omega_{e_1}\sigma_{ee}^1 + \omega_{e_2}\sigma_{ee}^2. \tag{2}$$

$$H_I = g_a(a_0^\dagger\sigma_1 + h\cdot c) + g_b(a_0^\dagger\sigma_2 + h\cdot c) \tag{3}$$

where $a_0^\dagger(a_0)$ is the creation (annihilation) operator for the photons on the $j$th single-mode cavity. Among them, $\sigma_{ee}^I = |e\rangle_I\langle e|(I = 1,2)$, $\sigma_L = |g\rangle_L\langle e|(L = 1,2)$ is the ladder operator for the $L$th quantum emitter, and $g_J(J = a,b)$ denotes the cavity-qubit coupling strength. Finally,

$$H_d = \lambda(\sigma_1^\dagger\sigma_2 + \sigma_2^\dagger\sigma_1) \tag{4}$$

describes the $DDI$, and $\lambda = |d|^2(1-3cos^2\theta)/|r|^3$ is coupling strength of the $DDI$, which can be controlled by the distance $r$ (the minimal average interatomic distance of $0.2\lambda_{wavelength}$ [33]; as the distance increases to $0.5\lambda_{wavelength}$ [19], the $DDI$ can be neglected.) between the two atoms and the angle $\theta$ between $r$ and the atomic dipole moment $d$.

Since the number of excitations is conserved in this hybrid system, for the one-excitation subspace, we assume the stationary eigenstate is

$$|\Psi(E)\rangle = \sum_j \alpha(j)a_j^\dagger|0\rangle|g\rangle + u_{e_1}|0\rangle|e_1\rangle + u_{e_2}|0\rangle|e_2\rangle, \tag{5}$$

where $|0\rangle$ represents the vacuum of the cavity fields, $\alpha(j)$ is the amplitude of the single-photon state in the $j$th resonator, and $u_{e_1}$ ($u_{e_2}$) is the probability of the amplitude of the state $|0\rangle|e_1\rangle$ ($|0\rangle|e_2\rangle$). The Schrödinger equation, $H|\Psi(E)\rangle = E|\Psi(E)\rangle$, and the scattering equations for a single photon with a discrete-coordinate representation can be used to obtain,

$$(E - \omega_c + V)\alpha(x) = -\zeta[\alpha(x-1) + \alpha(x+1)], \tag{6}$$

with the effective potential, $V(j) = [2\lambda g_a g_b + g_a^2(E-\omega_{e_2}) + g_b^2(E-\omega_{e_1})]/[(E-\omega_{e_1})(E-\omega_{e_2}) - \lambda^2]$.

Considering the interaction between the two-level atoms, one can observe either the transmitted wave or the reflected wave. The corresponding wave function can be expressed as

$$\alpha(j) = \begin{cases} e^{ikj} + re^{-ikj} & x<0, \\ te^{ikj} & x>0, \end{cases}$$

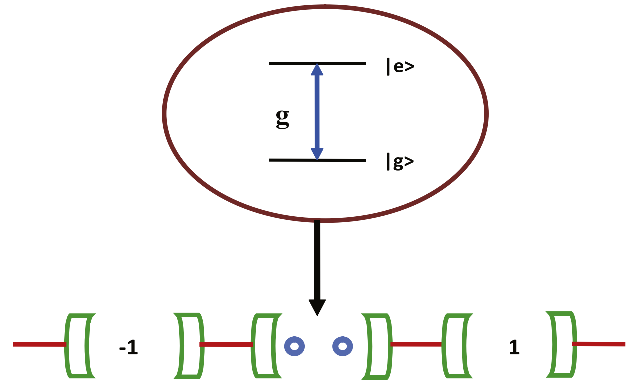


**Fig. 1.** Single-photon transport in a $1DCRW$ coupled to two-level atoms.

where $r$ and $t$ are the reflection and transmission amplitudes, respectively. In the $CRW$, the dispersion relation of the plane waves is

$$E = \omega_c - 2\zeta\cos k, \tag{7}$$

which is an energy-band structure. By solving the equation above, we can obtain the transmission and reflection amplitudes

$$t = \frac{2i\zeta\sin k}{2i\zeta\sin k + V}, \tag{8}$$

$$r = -\frac{V}{2i\zeta\sin k + V}. \tag{9}$$

We obtain the transmission coefficient, $T = |r|^2$. Eq. (8) shows that the transmission coefficient is a function of the atom frequency, the coupling strength and the wave vector of the incident photon. We know that the transmission vanishes when $k = 0, \pi or E = [(\omega_{e_1} + \omega_{e_2}) \pm \sqrt{(\omega_{e_1}-\omega_{e_2})^2 + 4\lambda^2}]/2$. Meanwhile, we further find that there is perfect transmission at $E = (g_a^2\omega_{e_2} + g_b^2\omega_{e_1} - 2\lambda g_a g_b)/(g_a^2 + g_b^2)$ due to the constructive interference.

## 3. Transmission spectra of the transporting photon

We now numerically analyze the transmission properties of single photons being transported along the $CRW$ with two-level atoms. The coupling strengths are important parameters in this hybrid system. Here, we show $T$ as a function of the energy $E$ with different $g$ values in Fig. 2(a) to show the influence of the coupling strengths on the transmission properties. Here, $g_a = g_b = g$ and $\lambda = 0$, which mean that the coupling strengths of the two-level atoms are equal when the $DDI$ is not considered. From Fig. 2(a), we can observe that as the coupling strengths increase, the maximum value of the transmission coefficient gradually decreases and the band width increases. The transmission spectrum is symmetrical around the energy $E = 2$. In Fig. 2(b), we show how the relevant transmission probability of the transporting photon depends on its energy and the variable coupling strength $g_b$. The numerical results suggest that when the coupling strength of the atom constantly changes, the phenomenon is basically the same as that in Fig. 2(a). This means that the transmission of the incident photon cannot be controlled by adjusting the coupling strengths of the two-level atoms. The coupling strengths can only change the frequency band width of the transmission spectrum.

To control the transmission of the resonant photon, we show that the $DDI$ affect the single photons scattering properties in the case of a pair of atoms coupled to the $CRW$. In Fig. 3(a), in contrast to Fig. 2(a), we observe that the $DDI$ can change the position of the dip and the transmission spectrum is not symmetric. In Fig. 3(b), we maintain the $DDI$ unchangeable but change the sizes of the coupling strengths. The typical transmission spectrum then splits into two dips. However, when