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The cubic–quintic–septic complex Ginzburg–Landau equation formulation of optical pulse propagation in 3D doped Kerr media with higher-order dispersions

Martin Djoko *, T.C. Kofane

Laboratory of Mechanics, Department of Physics, Faculty of Science, University of Yaounde I, P.O. Box 812, Yaoundé, Cameroon
Centre d'Excellence Africain en Technologies de l'Information et de la Communication (CETIC), University of Yaounde I, P.O. Box 812, Yaoundé, Cameroon

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A B S T R A C T

We investigate the propagation characteristics and stabilization of generalized-Gaussian pulse in highly nonlinear homogeneous media with higher-order dispersion terms. The optical pulse propagation has been modeled by the higher-order $(3+1)$-dimensional cubic–quintic–septic complex Ginzburg–Landau [$(3+1)$D CQS-CGL] equation. We have used the variational method to find a set of differential equations characterizing the variation of the pulse parameters in fiber optic-links. The variational equations we obtained have been integrated numerically by the means of the fourth-order Runge–Kutta (RK4) method, which also allows us to investigate the evolution of the generalized-Gaussian beam and the pulse evolution along an optical doped fiber. Then, we have solved the original nonlinear $(3+1)$D CQS-CGL equation with the split-step Fourier method (SSFM), and compare the results with those obtained, using the variational approach. A good agreement between analytical and numerical methods is observed. The evolution of the generalized-Gaussian beam has shown oscillatory propagation, and bell-shaped dissipative optical bullets have been obtained under certain parameter values in both anomalous and normal chromatic dispersion regimes. Using the natural control parameter of the solution as it evolves, named the total energy Q, our numerical simulations reveal the existence of 3D stable vortex dissipative light bullets, 3D stable spatiotemporal optical soliton, stationary and pulsating optical bullets, depending on the used initial input condition (symmetric or elliptic).

1. Introduction

The dynamics of solitons propagating in dispersive nonlinear media has been a major area of intense research activities, given its potential applicability in all optical communication systems. The soliton was first described by John Scott Russell [1] in 1834, who observed a solitary wave in the Union Canal, reproduced the phenomenon in a wave tank, and named it the "Wave of Translation". Several people contributed to the effort of trying to understand the phenomenon, including Airy [2], Boussinesq [3] and Korteweg and de Vries (KdV) [4], who in 1895 mathematically described weakly nonlinear shallow water waves with an equation that later came to be known as the KdV equation.

Since then, similar equations have been found in a wide range of physical phenomena, especially those exhibiting shock waves, traveling waves and solitons. In 1965, Zabusky and Kruskal [5] used a finite difference approach to numerically solve the KdV equation and the word "soliton" was first used. The soliton is a special subset of solitary waves that is stable to perturbations and mutual collisions.

In nonlinear optics, solitons can be classified as temporal [1-dimension (1D)] [6], spatial [1 and 2-dimensions (1D and 2D)] [7–9] or spatiotemporal [3-dimensions (3D)] [10] depending on whether the light is confined in time, space, or space and time, respectively. Propagation of optical pulses in monomode optical fibers is mainly influenced by the group velocity dispersion and the refractive index nonlinearity. Rapid progress in ultrashort time laser technology has made it possible that, optical pulses with durations comparable to the carrier oscillation cycle can be generated. The propagation of such ultrashort and intense pulses is then affected by additional physical mechanisms like self-steepening, Raman term and septic nonlinearities [11,12], where especially higher-order effects as fourth and sixth-order dispersion terms become important [13–17]. Temporal solitons in single mode optical fibers are the prototypical optical solitons. These were predicted theoretically in 1973 by Hasegawa and Tappert [18], and first observed experimentally in 1980 by Mollenauer et al. [19]. Extensive research since then has led to the current development of telecommunication systems based on solitons [20]. Compared to the

* Corresponding author.
E-mail addresses: djoko_martin2001@yahoo.fr (M. Djoko), tckofane@yahoo.com (T.C. Kofane).

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work on temporal solitons, progress in the area of multidimensional (spatial or spatiotemporal) optical solitons has been much slower. It has long been understood that self-focussing as a result of the Kerr nonlinearity could compensate for the spreading of a beam due to diffraction, but the resulting balance is unstable in dimensions greater than one [21], where the beam tends to diffract, collapse, or disintegrate into multiple filaments. Spatial solitons were first produced in liquid CS_2 [22,23], where an interference grating was employed to stabilize the solitons, and light filaments were observed [24] in resonant propagation through an atomic vapor, where the nonlinearity is saturable. 1D spatial solitons of the Nonlinear Schrödinger equation were generated in a glass waveguide in 1990 [25].

One of the major goals in the field of soliton physics is the production of light fields that are localized in all three dimensions of space as well as time, which we will refer to as 3D spatiotemporal solitons (STS) or light bullets. This results from the simultaneous balance of diffraction and group velocity dispersion (GVD) by self-focussing and nonlinear phase-modulation, respectively. In order to transmit ultrashort optical solitons at high bit rate in the picosecond and femtosecond regimes, several new effects such as sixth-order dispersion term, self-steepening (Kerr dispersion), gain, loss, spectral filtering, self-frequency shift (SFS) arising from stimulated Raman scattering and cubic, quintic, septic nonlinearities of dispersive and dissipative type are needed to be taken into account. Dissipative optical bullets, as a form preserving self-confined dissipative structures, have been described by the multidimensional standard complex cubic–quintic complex Ginzburg–Landau equations (CQGLE) [26,27]. Along the same line, Liu et al. [28] have investigated numerically the impact of phase on the collisions between solitary vortices in the frame of CQGLE. In fact, by gradually increasing initial kick, three generic outcomes have been identified: Merger of the two solitons into one, at small initial kick; Creation of an extra soliton, at intermediate initial kick; Quasi-elastic interactions at larger initial kick [28].

In general, equations describing soliton processes are usually obtained by certain approximating procedures affecting nonlinearity and dispersion. For example, picosecond pulses are well described by the NLS equation which account for second-order dispersion and self-phase modulation (SPM). However, it is known that the NLS equation does not give correct prediction for pulse width smaller than one picosecond. Thus, in extensive studies of ultrafast processes, the classical approximations often appear to be insufficient and higher-order effects become of importance. A typical example is high speed systems like nonlinear transmission lines in the femtosecond regime for soliton communications. To enlarge the information capacity, in ultrashort optical solitons at high bit rate in the picosecond and femtosecond regimes, we have taken into account that the presence of higher-order nonlinearities $\chi^{(5)}$, $\chi^{(7)}$, recently reported in chalcogenide glasses also suggests that materials appropriate to applications requiring higher-order nonlinearities exist. The nonlinear propagation of ultrashort pulses in a doped optical fiber is investigated with several new effects that we have taken into account such as the sixth-order dispersion term, self-steepening (Kerr dispersion), self-frequency shift (SFS) arising from stimulated Raman scattering and cubic, quintic, septic nonlinearities of dispersive and dissipative types which greatly influence their propagation properties. In cartesian coordinates, the governing equation that have been derived in the paraxial wave approximation and which describe light bullets, is based on the following higher-order (3+1)D cubic–quintic–septic complex Ginzburg–Landau (CQS-CGL) equation [29]

$$\begin{aligned} &i\psi_z+\frac{1}{4\beta_0}(\psi_{xx}+\psi_{yy})+p_r\psi_{\tau\tau}+q_r|\psi|^2\psi+c_r|\psi|^4\psi\\ &+s_r|\psi|^6\psi+\gamma_i\psi+d_{3i}\psi_{\tau\tau\tau}+d_{4i}\psi_{\tau\tau\tau\tau}\\ &+d_{5i}\psi_{\tau\tau\tau\tau\tau}+d_{6i}\psi_{\tau\tau\tau\tau\tau\tau}+(n_i)(|\psi|^2)_\tau\psi=Q. \end{aligned} \tag{1}$$

The right-hand side Q of Eq. (1) contains dissipative terms:

$$\begin{aligned} Q=\;&i\gamma_r\psi-iq_i|\psi|^2\psi-ic_i|\psi|^4\psi-ip_i\psi_{\tau\tau}\\ &-is_i|\psi|^6\psi+id_{3r}\psi_{\tau\tau\tau}+id_{4r}\psi_{\tau\tau\tau\tau}\\ &+id_{5r}\psi_{\tau\tau\tau\tau\tau}+id_{6r}\psi_{\tau\tau\tau\tau\tau\tau}+im_r(|\psi|^2\psi)_\tau, \end{aligned} \tag{2}$$

where the subscripts τ and z indicate the partial derivatives of ψ with respect to τ and z, respectively. The optical envelope $\psi(x,y,\tau,z)$ is a normalized complex function of four real variables x, y, τ, z, where x and y are the two transverse coordinates and z is the propagation distance. In studying the light bullets propagation, it is often convenient to measure time in the moving frame of the pulse optical envelope through the following transformation $\tau=t-\beta_{eff1}z$, that is, moving along with the pulse optical envelope $\psi(x,y,t,z)$, at the group velocity $v_g=\frac{1}{\beta_{eff1}}$, where t is the retarded time in the frame moving with the pulse.

In Eqs. (1) and (2), the parameters p_r, p_i, q_r, q_i, c_r, c_i, s_r, s_i, γ_r, γ_i, m_r, m_i, n_r, n_i, d_{3r}, d_{3i}, d_{4r}, d_{4i}, d_{5r}, d_{5i}, d_{6r} and d_{6i} are real constants, where p_r measures the wave dispersion, p_i the spectral filtering, q_r and q_i represent the Kerr nonlinearity coefficient and the nonlinear gain–absorption coefficient (if positive), respectively. The terms c_r and c_i stand for the saturation coefficient of the Kerr nonlinearity (if negative) and the saturation of the nonlinear-gain–absorption (if negative), respectively, while s_r and s_i represent the higher-order correction terms to the nonlinear refractive index and the nonlinear-gain–absorption, respectively. γ_r and γ_i stand for the coefficient for linear gain (if positive) and frequency shift, respectively. The quantities d_3, d_4, d_5, d_6 account for the third-order dispersion, fourth-order dispersion, fifth-order dispersion and sixth-order dispersion coefficients, respectively. m_r which is the nonlinear dispersion term, is responsible for self-steepening at the pulse edge. m_i describes the combined effect of nonlinear gain and/or absorption processes and is usually neglected in optical transmission system. n_i is responsible for the soliton self-frequency shift and n_r is also usually neglected in optical transmission system.

The paper is organized as follows. Section 2 is devoted to the presentation of the set of variational equations resulting from the Euler–Lagrange equations. In Section 3, we present a fully numerical simulation of the higher-order (3+1)D CQS-CGL equation which finally tests the variational approach. A good agreement between analytical and numerical methods is observed. Section 4 gives some concluding remarks.

2. Analytical treatment using variational approach

The obtained (3+1)D CQS-CGL equation, since it is not integrable, can be solved only numerically. However, some analytical approach is generally used. To analyze the solution's characteristics, we now consider a bell-type ansatz with a few free parameters which depend on the propagation distance z such as the amplitude, the temporal and spatial pulse widths, the position of the pulse maximum, the unequal wavefront curvatures, the chirp parameters, and the phase shift. In the following, we first use the variational method [30–32] for dissipative systems to obtain physical insight in terms of a few relevant parameters and then present numerical simulations that confirm the analytic predictions qualitatively. Very recently, variational approach has been used by Tang et al. [33] to predict that the Gaussian wave packet, in a helicoidal lattice model, performs anharmonic Bloch oscillations, which includes the fundamental harmonic and the high-order harmonics. Since the success of the above mentioned method lies in proper choice of trial function, we have chosen the generalized-Gaussian trial ansatz [31–34]

$$\begin{aligned} \psi(x,y,\tau,z)=\;&A(z)\exp(-\frac{\tau^2}{2T^2(z)}-\frac{\tau^4}{2T^4(z)}-\frac{x^2}{2X^2(z)}-\frac{x^4}{2X^4(z)}\\ &-\frac{y^2}{2Y^2(z)}-\frac{y^4}{2Y^4(z)}+i\theta(z)\tau^2+iC(z)x^2+i\sigma(z)y^2+i\phi(z)). \end{aligned} \tag{3}$$

The pulse evolution is described through the variation of the amplitude $A(z)$, temporal $T(z)$ and spatial $X(z)$ and $Y(z)$ pulse widths, unequal