Optics Communications 415 (2018) 31–38

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Tip-tilt disturbance model identification based on non-linear least squares fitting for Linear Quadratic Gaussian control

Kangjian Yang [a,b,c], Ping Yang [a,b], Shuai Wang [a,b], Lizhi Dong [a,b], Bing Xu [a,b,*]

[a] Key Laboratory on Adaptive Optics, Chinese Academy of Sciences, Chengdu 610209, China
[b] Institute of Optics and Electronics, Chinese Academy of Sciences, Chengdu 610209, China
[c] University of Chinese Academy of Sciences, Beijing 100049, China

ARTICLE INFO

Keywords:
Adaptive optics
Linear Quadratic Gaussian control
Vibration control
Model identification

ABSTRACT

We propose a method to identify tip-tilt disturbance model for Linear Quadratic Gaussian control. This identification method based on Levenberg–Marquardt method conducts with a little prior information and no auxiliary system and it is convenient to identify the tip-tilt disturbance model on-line for real-time control. This identification method makes it easy that Linear Quadratic Gaussian control runs efficiently in different adaptive optics systems for vibration mitigation. The validity of the Linear Quadratic Gaussian control associated with this tip-tilt disturbance model identification method is verified by experimental data, which is conducted in replay mode by simulation.

1. Introduction

Adaptive optics (AO) is a real-time correction system for wave-front distortions. AO systems are widely used in astronomical telescopes to compensate the atmospheric perturbation [1,2] and laser systems for high beam qualities [3–5]. The tip-tilt disturbances, which are the main components of wave-front distortions [6], are very detrimental to the performance of AO systems. As the complexity of AO systems is increasing, tip-tilt disturbances originate not only from atmospheric perturbation, but also from vibrations caused by system components such as cryo-coolers, pumps, fans and motors [7–10]. For instance, a 25% loss of Strehl Ratio (SR) could be attributed to vibrations on NAOS [8] while a 20 mas RMS jitter due to vibrations was estimated on Altair [10]. In order to achieve high image qualities or high beam qualities, tip-tilt disturbances affected by vibrations must be well corrected.

Linear Quadratic Gaussian (LQG) is an appealing control strategy for AO systems to mitigate tip-tilt disturbances affected by vibrations [11,12]. LQG control is an optimal correction law with respect to minimum residual phase variance, which performs an optimal state estimation thanks to a Kalman filter. Since the first time that Paschall and Anderson [13], and then B. Le Roux et al. [14,15] introduced Kalman filter in MultiConjugate Adaptive Optics (MCAO), there is a growing attention paid to LQG control based on Kalman filter. And LQG control strategy has been adapted in several modern AO systems with a significant gain on tip-tilt disturbances correction. The first laboratory validation of vibration filtering with LQG control for adaptive optics was carried out in 2008 [16], where the artificial vibration was almost suppressed. In 2012, the first on-sky single conjugated AO (SCAO) validation of full LQG control on the CANARY pathfinder was conducted [17], where about 10% to 20% increase of SR was achieved by vibration mitigation. The effectiveness of the tip-tilt disturbances correction with LQG control was also found in Gemini Planet Imager [18].

LQG control achieves optimal tip-tilt disturbance correction when provided with precise tip-tilt disturbance model [19]. It is essential to identify exactly tip-tilt disturbance model from the wave-front sensor (WFS) measurements. But tip-tilt disturbances caused by vibrations are difficult to be characterized, because they vary from one system to another as well as evolve in time along observation. The online identification method may be a good choice. Nowadays, time domain methods such as prediction error method (PEM), sub-space identification (SSI) and extended Kalman filter (EKF) are introduced to identify model parameters for LQG control [20,21]. Although, the non-convexity of the criterion of PEM makes it not easy to optimize due to several local minima. For the SSI, it is not proper to identify model for obtaining the meaningful physical model parameters. Besides, based on the physical model structure of vibrations, the EKF is sensitive to the initialization values, leading to poor performance or instability of the closed loop. In frequency domain, an online model parameters identification method

* Correspondence to: Box 350, Shuangliu, Chengdu, Sichuan, China.
E-mail address: bing_xu_ioe@163.com (B. Xu).

https://doi.org/10.1016/j.optcom.2018.01.022
Received 28 July 2017; Received in revised form 10 January 2018; Accepted 11 January 2018

has been proposed [22]. This method uses maximum likelihood method to identify model parameters, and 80%–90% energy of the vibrations is mitigated by LQG control. However, the minimum and maximum frequencies between which the vibration peaks exist need to be carefully chosen beforehand according to each AO system.

In this paper, we propose a method to identify vibration model parameters based on the spectrum of measurement data for LQG control. It is an online model parameters identification method which fits the disturbance spectrum with the Levenberg–Marquardt method. The Levenberg–Marquardt method is an iterative method with low computational budget. Also, the isolation procedure proposed here can pick vibration peaks accurately. Besides the information about the noise frequency and the model structure of vibrations, it runs successfully on experimental data without any other prior information about disturbances. This identification method makes it easy that vibration mitigation achieves with LQG control, where about 90% energy of the vibrations is mitigated. The analysis of the on-line identification and the robustness is also presented below in this paper.

We organize the paper as follows. In Section 2, we briefly provide the state–space description of AO system and the formalism of LQG control. In Section 3, our model identification method is presented. Then in Section 4, we present the results of the tip-tilt disturbances identification and correction. Finally, discussion and conclusions are presented in Section 5.

2. LQG control for tip-tilt disturbance correction

LQG control is a globally optimal strategy for tip-tilt disturbances correction and fully described in [23]. We recall, as in the reference, that tip-tilt mirror can be regarded as a linear system and that there is a two-frame loop delay. These assumptions for LQG control are generally valid for current AO systems. Besides, the tip-tilt mirror response is considered as instantaneous here with respect to the loop sampling period (or frame) T. So, the state–space representation of the measurement equation can be described as follows:

$$y_n = CX_n - DNu_{n-2} + w_n, \tag{1}$$

where C, D and N denote measurement matrix, WFS characteristic matrix and the tip-tilt mirror (TTM) influence matrix respectively. y_n is the WFS measurements acquired between $(n-1)T$ and nT which denotes the noisy measurement of the residual phase averaged over $[(n-2)T, (n-1)T]$. The tip-tilt disturbances phase φ_n and φ_{n-1} which, respectively, denote the perturbation phase averaged over $[(n-1)T, nT]$ and $[(n-2)T, (n-1)T]$ composite the state X_n. The measurement matrix C, namely the observation matrix, denotes the relation between the state X_n and the WFS measurements. In other words, the relation can be expressed as below:

$$X_n = \begin{bmatrix} \varphi_n \\ \varphi_{n-1} \end{bmatrix}, \tag{2}$$

$$y_n = \underbrace{\begin{bmatrix} 0 & D \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} \varphi_n \\ \varphi_{n-1} \end{bmatrix}}_{X_n} - DNu_{n-2} + w_n. \tag{3}$$

u_{n-2} is the control voltage of the TTM with respect to the correction phase Nu_{n-2}, which is constant with a zero-order hold during $[(n-2)T, (n-1)T]$, and w_n is the measurement noise assumed to be zero-mean white Gaussian noise with a variance Σ_w.

Besides, the tip-tilt disturbance temporal model can be described by state–space vector X_n, as presented below:

$$X_{n+1} = AX_n + V_n, \tag{4}$$

where A denotes the model parameters matrix, which defines the statistical characteristics of the tip-tilt disturbances, and V_n is zero-mean white Gaussian noise with a covariance matrix Σ_v.

With Eqs. (1)–(4), the state–space description of AO system is acquired. Then the LQG control based on Kalman filter can be conducted, which is shown in Eqs. (5–7) [24]:

$$\hat{X}_{n/n} = \hat{X}_{n/n-1} + H_\infty(y_n - C\hat{X}_{n/n-1} + DNu_{n-2}), \tag{5}$$

$$\hat{X}_{n+1/n} = A\hat{X}_{n/n}, \tag{6}$$

$$u_n = P\hat{X}_{n+1/n}, \tag{7}$$

where $\hat{X}_{n/n}$ is the estimation vector of X_n obtained by using all the measurements until nT. Tip-tilt correction is deduced thanks to a classis least-square projection of the predicted disturbance phase onto the TTM. The matrix $P = N^{-1}(1, 0)$ extracts the predicted disturbance phase from the predicted state vector and then projects it on the TTM to obtain the control voltage. This amounts to the following expression for u_n:

$$u_n = N^{-1}\hat{\varphi}_{n+1/n}. \tag{8}$$

H_∞ denotes the asymptotic gain of Kalman filter, which is defined as:

$$H_\infty = \Sigma_\infty C^T(C\Sigma_\infty C^T + \Sigma_w)^{-1}, \tag{9}$$

where Σ_∞ is the asymptotic solution of a Riccati equation computed as below [24]:

$$\Sigma_\infty = A\Sigma_\infty A^T + \Sigma_v - A\Sigma_\infty C^T(C\Sigma_\infty C^T + \Sigma_w)^{-1}C\Sigma_\infty A^T. \tag{10}$$

The asymptotic gain of the Kalman filter is independent from measurements which can be computed off-line. From Eqs. (5) and (6), LQG control law performs the tip-tilt disturbances estimation via a Kalman filter. The key issue is to build the filter with correct parameters, i.e., with a correct temporal model of the tip-tilt disturbances.

3. Tip-tilt disturbance model identification based on spectrum

3.1. Temporal model of tip-tilt disturbances

Vibrations which cause tip-tilt disturbances are mainly from system components such as cryo-coolers, fans and motors. Therefore, one vibration can be described by a dampened oscillatory signal φ^{vib} which is generated by a forcing function ξ at the pulsation $\omega_0 = 2\pi f_{vib}$ (natural frequency f_{vib}) [16]. The dampened oscillatory signal φ^{vib} can be described by the second-order differential equation:

$$\ddot{\varphi}^{vib} + 2K\omega_0\dot{\varphi}^{vib} + \omega_0{}^2\varphi^{vib} = G\omega_0{}^2\xi, \tag{11}$$

where K is the damping coefficient and G is the static gain. In discrete-time description, the dampened oscillatory signal φ^{vib} can be described by the second-order Auto-Regressive (AR2) model [16]:

$$\varphi_n^{vib} = a_1\varphi_{n-1}^{vib} + a_2\varphi_{n-2}^{vib} + v_n, \tag{12}$$

where v_n is a zero-mean white Gaussian noise and the coefficients a_1, a_2 are defined by

$$a_1 = 2e^{-K\omega_0 T}\cos(\omega_0 T\sqrt{1-K^2}), \tag{13}$$

$$a_2 = -e^{-2K\omega_0 T}. \tag{14}$$

The damping coefficient K is related to the vibration bandwidth. In practice, the tip-tilt disturbances are affected by multiple vibrations from different system components. The temporal model of the tip-tilt disturbances can be described by a sum of dampened oscillatory signals which are interpreted with the AR2 model [22]:

$$\varphi_n = \varphi_n^{vib,1} + \varphi_n^{vib,2} + \cdots + \varphi_n^{vib,m}, \tag{15}$$

$$\varphi_n^{vib,i} = a_1^{vib,i}\varphi_{n-1}^{vib,i} + a_2^{vib,i}\varphi_{n-2}^{vib,i} + v_n^{vib,i}. \tag{16}$$

In fact, disturbances caused by atmospheric perturbation can also be described by the dampened oscillatory signal [22]. Then it is followed