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Signal-to-noise analysis of a birefringent spectral zooming imaging spectrometer

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ABSTRACT

Study of signal-to-noise ratio (SNR) of a novel spectral zooming imaging spectrometer (SZIS) based on two identical Wollaston prisms is conducted. According to the theory of radiometry and Fourier transform spectroscopy, we deduce the theoretical equations of SNR of SZIS in spectral domain with consideration of the incident wavelength and the adjustable spectral resolution. An example calculation of SNR of SZIS is performed over 400–1000 nm. The calculation results indicate that SNR with different spectral resolutions of SZIS can be optionally selected by changing the spacing between the two identical Wollaston prisms. This will provide theoretical basis for the design, development and engineering of the developed imaging spectrometer for broad spectrum and SNR requirements.

1. Introduction

Spectral resolution is one of the critical parameters of an imaging spectrometer. However, in practical applications, different spectral resolution is often required for the imaging spectroscopy. Traditional fixed spectral resolution and the corresponding data cubes always bring out redundant and not necessary results. Spectral zooming imaging spectrometer (SZIS) developed by the authors in 2013 [1] is a novel scheme of Fourier transform imaging spectrometer (FTIS). It not only has advantages similar to other FTISs' like superior throughput, simultaneous measurement of all frequencies, but also retains compact structure, and spectral resolution zooming capability. It provides an effective and attractive scheme to solve the problem by a compact static birefringent imaging spectrometer based on double Wollaston prisms with an adjustable air-gap between them to gain spectral zooming capability. Signal-to-noise ratio (SNR) is another key parameter to describe the performance of an imaging spectrometer. To get high quality spectral images, the SNR must be fine [2]. In other words, it is difficult to obtain useful data while the incident radiation is much less than the noise for a high spectral resolution device. Therefore, to know the exact performance of the SZIS, SNR analysis is very meaningful.

In the past ten years, SNR of variety of reflective interferometric FTISs were estimated and tested. Sellar and Boreman compared relative SNR of different spectral and scan classes of imaging spectrometer and presented a general SNR estimation of static interferometric imaging spectrometer (windowing mode) [3]. Barducci et al. analyzed SNR of a Sagnac interferometer based FTIS in different wavelengths [4]. Bergstrom et al. proposed a test result of a static corner-cube Michelson FTIS' SNR in different spectral resolution in long wave infrared band [5]. However, since the structure's differences and existing of the polarizers and analyzers, the SNR of the birefringent FTISs are quite different. Zhang et al. estimated SNR with points of interferogram of a Savart polariscope FTIS [6]. The drawback is that the SNR of interferogram cannot characterize the instrument's performance directly and make the comparison with other kind of imaging spectrometers much more complicated. Besides, spectral data cube is always gained through Fourier transform from interferogram data which FTIS records [7]. This kind of frequency transformation would often cause variations of signal and noise. And this process often ignores the influence of pixels' inconsistency of CCD detector. Considering these factors, our work mainly focuses on the SNR in spectral domain. Quantitative calculation of SNR is implemented to accurately display each band performance of the spectral image obtained from the SZIS. Relationship between spectral resolution and SNR is also shown.

2. System mode

Firstly, we briefly recall the operation principle of the SZIS [1]. Its optical schematic is illustrated in Fig. 1. It consists of fore-optics

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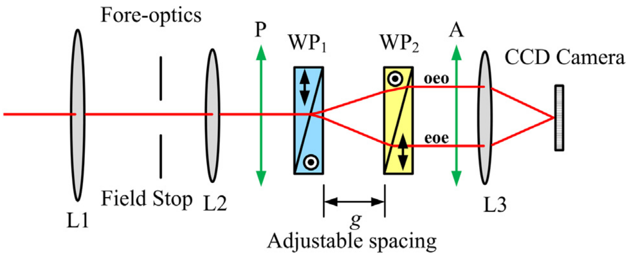

Fig. 1. Schematic of the optical system SZIS [1].

(L1, field stop and L2), a polarization interferometer (polarizer P, two identical Wollaston prisms WP$_1$ and WP$_2$, and analyzer A), an imaging lens L3 and a CCD camera. Light from objects is collected and collimated by fore-optics and then incident on P. The linearly polarized light from polarizer P is split into two beams by WPs. The two component rays (oeo and eoe) orthogonally polarized have a small lateral displacement between them with equal intensity. After passing through A, the two rays are resolved into linearly polarized light with the same vibration direction. Then they are interference on the focal plane of L3 and recorded by the focal plane array (FPA) of CCD camera. It should be noted that the developed system has no slit and works in windowing mode. Its spectral resolution zooming capability is achieved by adjusting the spacing between WP$_1$ and WP$_2$.

The signal-to-noise ratio is defined as the ratio of signal power to the noise power. To calculate SNR in spectral domain, first we obtain the numerical value of signal and noise recorded in each pixel from interferogram data cube. The signal is the actual output described in the form of photoelectrons. The noise is the random fluctuation in spatial dimension operated in the form of standard deviation. Through Fourier transform, both signal and noise in the spectral domain can be obtained. Also SNR of the developed SZIS can be acquired for each spectral interval.

3. Interferogram and reconstructed spectrum

For interferogram analysis, we considered the influence of extinction ratio of polarizer due to the crystal dichroism. In ideal condition, the two polarizers at 45° to the optic axes of prisms: P polarizes the light and A recombines the two beams into an identical polarization direction. Actually, part of light still gets through at 135° to the optic axes. It is indicated that the polarization devices are not ideal in practical application. Natural light emerging from polarization devices will become partially polarized light that can be resolved into two directions. The intensity at 45° and 135° are written as $I_{\max}$ and $I_{\min}$ respectively. Therefore, extinction ratio of the polarizer P can be defined as:

$$\eta_P = \frac{I_{\min}}{I_{\max}}. \tag{1}$$

Similarly, analyzer A's extinction ratio is defined as η_A.

WP$_1$ and WP$_2$ resolve incident linearly polarized light into two beams respectively along the x-axis and y-axis direction. Therefore, $I_{\max}$ and $I_{\min}$ can also be resolved into the x and y directions. Suppose the incident amplitude is E_0. When the light passes through P, E_0 will be resolved into $E_v = E_0\left(\cos 45^\circ - \sqrt{\eta_P}\sin 45^\circ\right)$ in the x direction and $E_p = E_0\left(\sin 45^\circ + \sqrt{\eta_P}\cos 45^\circ\right)$ in the y direction as shown in Fig. 2. Hence the corresponding intensity transmittance can be given as $T_v = \left(\cos 45^\circ - \sqrt{\eta_P}\sin 45^\circ\right)^2$ and $T_p = \left(\sin 45^\circ + \sqrt{\eta_P}\cos 45^\circ\right)^2$. According to Malus' law, the transmittances of the two linearly polarized lights oeo and eoe passing through analyzer A are $T_{voeo} = \cos^2 45^\circ$ and $T_{peoe} = \sin^2 45^\circ$ respectively, as shown in Fig. 3.

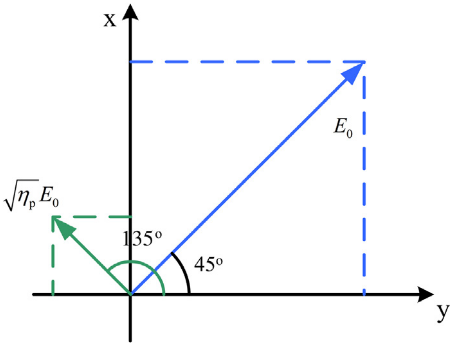

Fig. 2. The linearly polarized light passing through polarizer resolved into x and y direction.

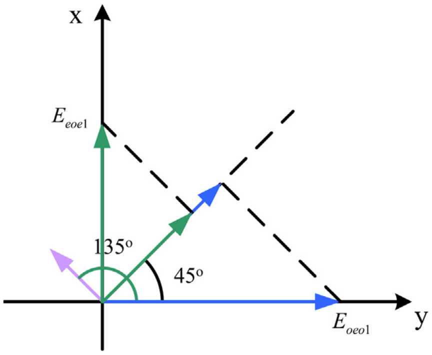

Fig. 3. oeo and eoe light passing through analyzer resolved into 45° and 135°.

According to the superposition principle of waves, the detected intensity on FPA is:

$$\begin{aligned} I(\Delta,\sigma) &= \left(E_o + E_e\right)\cdot\left(E_o + E_e\right)^* \\ &= E_0^2 T_{olight} + E_0^2 T_{elight} + 2E_0^2\sqrt{T_{olight}T_{elight}}\cos(2\pi\sigma\Delta), \end{aligned} \tag{2}$$

with the transmittances of ordinary and extraordinary rays

$$T_{olight} = T_v T_P T_W T_W T_A T_{voeo}, \tag{3}$$

$$T_{elight} = T_p T_P T_W T_W T_A T_{peoe}. \tag{4}$$

Here $\Delta = z_o - z_e$ is the optical path difference (OPD) introduced by WPs, and σ is the spectral wavenumber. T_P, T_W and T_A respectively denotes transmittance of polarizer P, Wollaston prism and the analyzer A.

For the imaging spectrometer, optical path difference can be rewritten as:

$$\Delta = d\sin i, \tag{5}$$

where i is the incident angle and d is the lateral displacement between the two polarized rays from the double WPs. Ray tracing of the polarization interferometer is illustrated in Fig. 4.

The lateral displacement d generated by the double-Wollaston prisms with zooming spacing g can be expressed as [1]:

$$d = \frac{t\tan(\phi_{1oe}-\theta) + g\tan\alpha_2}{1-\tan\theta\cdot\tan(\phi_{1oe}-\theta)} + \frac{t\tan(\theta-\phi_{1eo}) + g\tan\alpha_4}{1-\tan\theta\cdot\tan(\theta-\phi_{1eo})}, \tag{6}$$

with the parameters

$$\phi_{1oe} = \arcsin\left(\frac{n_o\sin\theta}{n_e}\right), \tag{7}$$

$$\phi_{1eo} = \arcsin\left(\frac{n_e'(\theta_1)\sin\theta}{n_o}\right), \tag{8}$$