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Enhancing optical nonreciprocity by an atomic ensemble in two coupled cavities

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ABSTRACT

We study the optical nonreciprocal propagation in an optical molecule of two coupled cavities with one of them interacting with a two-level atomic ensemble. The effect of increasing the number of atoms on the optical isolation ratio of the system is studied. We demonstrate that the significant nonlinearity supplied by the coupling of the atomic ensemble with the cavity leads to the realization of greatly-enhanced optical nonreciprocity compared with the case of single atom.

1. Introduction

Investigation on the realization of optical nonreciprocity has achieved significant interest for its applications on constructing quantum networks [1,2]. Optical isolator is a typical nonreciprocal device breaking the Lorentz reciprocity with an asymmetric scattering matrix [3,4], and allows the signal to transport in one direction but not in the opposite direction [5].

As typical examples, the realization of optical nonreciprocity via magneto materials [6–11], moving photonic crystals [12,13], quasi-two-dimensional metasurfaces [14], optomechanical systems [15–21], and nonreciprocal diffraction systems with photonic Aharonov–Bohm effects [22], are fundamentally based on breaking time-reversal symmetry. On the other hand, the nonlinear interaction [23] in an optical system can also result in the optical nonreciprocity. For example, the Raman amplification [24,25], nonlinear micro-resonator with asymmetric structure [26], nonlinear metasurfaces or massless chiral Dirac fermions with second-order susceptibility tensor [27,28], and the $\chi^{(3)}$ Kerr interaction, are used to generate the nonreciprocity [29–31]. In practical applications, the above two kinds of mechanisms are usually combined together to implement controllable and enhanced optical nonreciprocity [32].

Recently, a scheme with high isolation ratio has been studied subsequently in a hybrid system consisting of two coupled cavities (an optical molecule [33–37]) with one of them coupling with a single two-level atom [38]. In such a scheme, the atom–cavity coupling induces the nonlinear terms in the evolution equation, and as a result leads to a high isolation ratio for the optical propagation. That is, the transmission rate for the optical probe field in one direction is much larger than that in the opposite direction. Since the isolation ratio depends strongly on the strength of the coupling between the atom and the cavity, a high isolation ratio requires strong atom–cavity coupling, which limits the realistic applications of optical nonreciprocity in such systems.

In order to break the above limitation and improve the isolation ratio, we propose an optical nonreciprocal scheme by replacing the single atom by an ensemble of N atoms in the model proposed in Ref. [38]. By defining the collective operators of the atomic ensemble, which satisfy the SU(2) algebra, we get the quantum Langevin equations for the cavity operators and atomic collective operators in the system. By means of the mean-field approximation, we can obtain the steady state solutions with the assistance of numerical calculation. Based on the steady-state solutions, we investigate the isolation ratio for the optical probe field incident from different directions in the proper region of parameters, and find that the atomic ensemble is beneficial to enhance the nonlinear effect, leading to a much more significant optical nonreciprocity compared with the case of single atom. Moreover, we show an obvious enlargement of the isolation ratio by increasing the number of atoms in the ensemble. We would like to point out that the mean field approximation used here and in e.g. Ref. [38], may be not always reasonable especially for the case of a few atoms. However, in our case of atomic ensemble, the steady state mean values for the atomic

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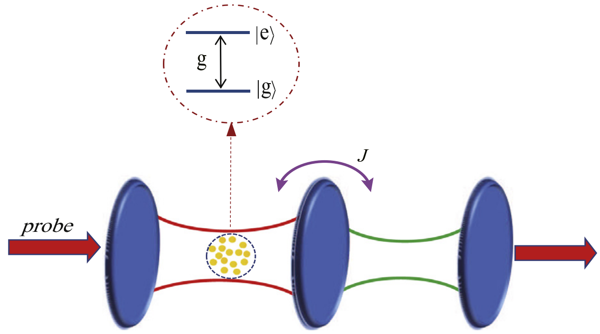

Fig. 1. (Color online). Schematic diagram of two coupled cavities interacted with a two-level atomic ensemble in the first cavity. An optical probe field is incident on the first cavity from the left side.

collective operators are much larger than 1, and guarantee the validity of the mean field approximation.

We organize the paper as follows. In Section 2, we introduce the Hamiltonian and quantum Langevin equations of the system of the optical molecule coupled with the atomic ensemble. The optical non-reciprocal response is discussed numerically in Section 3 based on the steady-state solutions. Finally, a brief conclusion is drawn in Section 4.

2. The Hamiltonian and steady state

The system under consideration is shown in Fig. 1. Two single-mode cavities with the frequencies of ω_1 and ω_2 are directly coupled at rate J. The first cavity is driven by a probe field with ω_p and ε_p being the corresponding frequency and amplitude, respectively. The atomic ensemble of N two-level atoms with transition frequency ω_e in the first cavity is coupled to the cavity with the single-atom coupling strength g.

Under the rotating-wave approximation and electric-dipole approximation, we obtain the Hamiltonian of such a system as ($\hbar = 1$)

$$\begin{aligned} H =\ & \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \frac{\omega_e}{2}\sum_i \sigma_z^{(i)} \\ & + J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) + ig\left(a_1 \sum_i \sigma_{eg}^{(i)} - a_1^\dagger \sum_i \sigma_{ge}^{(i)}\right) \\ & + i\sqrt{\kappa_1^e}\left(e^{-i\omega_p t}\varepsilon_p a_1^\dagger - e^{i\omega_p t}\varepsilon_p^* a_1\right), \end{aligned} \tag{1}$$

where a_j ($a_j^\dagger$, $j = 1, 2$) is the bosonic annihilation (creation) operator of the cavity mode j. $\sigma_{ge}^{(i)}$ $\left(\sigma_{eg}^{(i)}\right)$ is the transition operator between the states $|e\rangle$ and $|g\rangle$ of the ith atom, and $\sigma_z^{(i)} = \sigma_{ee}^{(i)} - \sigma_{gg}^{(i)}$ represents the component of Pauli operator along z-axis. κ_1^e denotes the coupling between the first cavity and the probe field.

Usually, the above Hamiltonian is dealt by performing the Holstein–Primakof (HP) transformation under the low-excitation approximation [39] to illustrate the motion of the atomic ensemble as a single bosonic mode. However, to investigate the nonlinearity induced by the coupling between the atomic ensemble and cavity, we here resort to a numerical solution beyond the HP transformation with the low-excitation approximation. To this end, we introduce the collective operators for the atomic ensemble

$$L_- = \sum_{i=1}^N \sigma_{ge}^{(i)},\ L_+ = \sum_{i=1}^N \sigma_{eg}^{(i)},\ L_z = \frac{1}{2}\sum_{i=1}^N \sigma_z^{(i)}, \tag{2}$$

which satisfy the SU(2) algebra with the commutation relation

$$\left[L_z, L_\pm\right] = \pm L_\pm,\ \left[L_+, L_-\right] = 2L_z. \tag{3}$$

Then in a rotating frame with respect to the frequency of probe field ω_p, we obtain the following Hamiltonian

$$\begin{aligned} \mathcal{H} =\ & \Delta_1 a_1^\dagger a_1 + \Delta_2 a_2^\dagger a_2 + \Delta_e L_z \\ & + J\left(a_1^\dagger a_2 + a_2^\dagger a_1\right) + ig\left(a_1 L_+ - a_1^\dagger L_-\right) \\ & + i\sqrt{\kappa_1^e}\left(\varepsilon_p a_1^\dagger - \varepsilon_p^* a_1\right), \end{aligned} \tag{4}$$

where $\Delta_1 = \omega_1 - \omega_p$, $\Delta_2 = \omega_2 - \omega_p$ and $\Delta_e = \omega_e - \omega_p$ are the detunings of the two cavities and atomic transition from the probe field, respectively.

The corresponding quantum Langevin equations (QLEs) based on Hamiltonian (4) is

$$\begin{aligned} \dot{a}_1 =\ & -i\Delta_1 a_1 - iJa_2 - gL_- + \sqrt{\kappa_1^e}\varepsilon_p \\ & - \frac{\kappa_1^o}{2}a_1 - \frac{\kappa_1^e}{2}a_1 + \sqrt{\kappa_1^o}a_{1,\text{in}}^o + \sqrt{\kappa_1^e}a_{1,\text{in}}^e, \end{aligned} \tag{5a}$$

$$\begin{aligned} \dot{a}_2 =\ & -i\Delta_2 a_2 - iJa_1 - \frac{\kappa_2^o}{2}a_2 - \frac{\kappa_2^e}{2}a_2 \\ & + \sqrt{\kappa_2^o}a_{2,\text{in}}^o + \sqrt{\kappa_2^e}a_{2,\text{in}}^e, \end{aligned} \tag{5b}$$

$$\dot{L}_z = ga_1 L_+ + ga_1^\dagger L_- - \frac{\gamma}{2}\left(2L_z + N\right) + \sqrt{\gamma}L_{z,\text{in}}, \tag{5c}$$

$$\dot{L}_- = -i\Delta_e L_- - 2ga_1 L_z - \frac{\gamma}{2}L_- + \sqrt{\gamma}L_{-,\text{in}}. \tag{5d}$$

Here κ_j^e (κ_j^o) refers to the coupling rate between the jth cavity and the input or output field (the other remaining loss rate for the jth cavity), with $a_{j,\text{in}}^e$ ($a_{j,\text{in}}^o$) the corresponding quantum input noise term of the cavity with zero mean value. $L_{z,\text{in}}$ and $L_{\pm,\text{in}}$ with zero mean values are the noise operators of the atoms. γ is the atomic decay rate. By considering the mean-field assumption, we get the averaged QLEs under steady state as

$$0 = -i\Delta_1\alpha_1 - iJ\alpha_2 - g\xi + \sqrt{\kappa_1^e}\varepsilon_p - \frac{\kappa_1^o}{2}\alpha_1 - \frac{\kappa_1^e}{2}\alpha_1, \tag{6a}$$

$$0 = -i\Delta_2\alpha_2 - iJ\alpha_1 - \frac{\kappa_2^o}{2}\alpha_2 - \frac{\kappa_2^e}{2}\alpha_2, \tag{6b}$$

$$0 = g\alpha_1\xi^* + g\alpha_1^*\xi - \frac{\gamma}{2}\left(2\xi_z + N\right), \tag{6c}$$

$$0 = -i\Delta_e\xi - 2g\alpha_1\xi_z - \frac{\gamma}{2}\xi, \tag{6d}$$

where we have already considered the time differential of the operators as zero and replaced all the operators by their averages. In addition, we have defined $\langle a_j\rangle = \alpha_j$ ($j = 1, 2$), $\langle L_-\rangle = \xi$, $\langle L_+\rangle = \xi^*$, and $\langle L_z\rangle = \xi_z$. Note that the mean values of all the noise operators are zero.

Using the input–output relation

$$a_{2,\text{out}}^e + a_{2,\text{in}}^e = \sqrt{\kappa_2^e}a_2, \tag{7}$$

one can get the corresponding transmitted field amplitude S_{out}^R, which is equivalent to the output field from the second cavity in this case

$$S_{\text{out}}^R = \langle a_{2,\text{out}}^e\rangle = \sqrt{\kappa_2^e}\alpha_2, \tag{8}$$

and the value of α_2 can be solved numerically according to Eq. (6).

If the propagation direction of the probe field is opposite, that is, the probe field is incident on the second cavity from the right side, the QLEs of the system under steady state are

$$0 = -i\Delta_1\alpha_1 - iJ\alpha_2 - g\xi - \frac{\kappa_1^o}{2}\alpha_1 - \frac{\kappa_1^e}{2}\alpha_1, \tag{9a}$$

$$0 = -i\Delta_2\alpha_2 - iJ\alpha_1 - \frac{\kappa_2^o}{2}\alpha_2 - \frac{\kappa_2^e}{2}\alpha_2 + \sqrt{\kappa_2^e}\varepsilon_p, \tag{9b}$$

$$0 = g\alpha_1\xi^* + g\alpha_1^*\xi - \frac{\gamma}{2}\left(2\xi_z + N\right), \tag{9c}$$

$$0 = -i\Delta_e\xi - 2g\alpha_1\xi_z - \frac{\gamma}{2}\xi, \tag{9d}$$

which are similar to Eqs. (6a)–(6d) except the term for the input probe field.