Optics Communications 415 (2018) 164–167

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

The extent to which path-integral models account for evanescent (tunneling) and complex (near-field) waves

Anedio Ranfagni, Daniela Mugnai, Ilaria Cacciari *

Istituto di Fisica Applicata "Nello Carrara", Consiglio Nazionale delle Ricerche, Via Madonna del Piano 10, 50019 Sesto Fiorentino, Italy

A R T I C L E   I N F O

Keywords:
Tunneling
Stochastic approach
Feynman's transition elements
Microwave optics

A B S T R A C T

The usefulness of a stochastic approach in determining time scales in tunneling processes (mainly, but not only, in the microwave range) is reconsidered and compared with a different approach to these kinds of processes, based on Feynman's transition elements. This latter method is found to be particularly suitable for interpreting situations in the near field, as results from some experimental cases considered here.

1. Introduction

Tunneling and related processes have continued to attract the interest of researchers, both from a theoretical point of view and also from experimental and applicative ones, as can be evidenced also in the recent literature [1–7].

In this context, of particular interest are the definition and the determination of the time scales of the processes considered, such as the decay-time of metastable states, the traversal-time of the barrier and, more generally, the time duration not only of classically forbidden processes (tunneling) but also those allowed classically in particular situations (near-field) which are, in a sense, comparable to cases of tunneling [8]. A field that has been shown to be particular useful for this kind of investigation is that of microwaves [9]. There, tunneling situations involve evanescent waves, as e.g. in sub-cutoff waveguides, while the propagation in the near field involves complex waves. In both cases, we can observe superluminal behaviors in wave propagation, a phenomenon to which a definitive interpretation has not yet been completely given [10].

A powerful method for dealing with these kinds of problems, in which dissipative effects play a crucial role, is represented by a stochastic model such as the one early formulated by Kac [11], then developed by De Witt-Morette and Foong [12], and subsequently applied to tunneling in order to interpret experimental results in the microwave range [13].

Another important approach to the same kind of problems is represented by the transition-elements by Feynman [14], which consist of weighted averages of the considered quantity, where the weighting function is $\exp(iS/\hbar)$, S being the action integral of the system under test. In our case, the application of this procedure to an evaluation of the time-duration of tunneling and wave propagation is given in [15].

The purpose of the present work is to establish a close correspondence and a comparison between the two approaches that have as their objectives the application to tunneling and to wave propagation in the near field. In Section 2 we analyze the tunneling processes in relation to a stochastic model, while Section 3 is devoted to a transition-element analysis of near-field propagation.

2. Tunneling as a stochastic process

Let us now briefly summarize the salient features of the stochastic model for tunneling [16]. In addition to the ordinary time t, we have to consider a "randomized time" $r \leqslant t$ [12], the density distribution of which $g(r,t)$ is given, in its asymptotic form, as a Gaussian [17,18]

$$g(r,t) \simeq \sqrt{\frac{a}{2\pi t}}\exp\left(\frac{-ar^2}{2t}\right), \tag{1}$$

where a is the dissipative parameter entering the telegrapher's equation, which is interpreted as a stochastic process [11,12].

When evanescent (tunneling) or complex waves (near field) are considered, the role of r and t are exchanged in accordance with the *ansatz* hypothesized in [18]. In this way, a plausible description of the experimental results can be obtained, including superluminal behaviors in microwave and optics [19]. A full justification of this procedure can be obtained when this class of processes is considered as a case of "weak measurement". Although the relative theory was originally proposed for quantum-mechanical situations [20], it has received considerable attention thanks to its ability to interpret also a variety of classical-physical situations [21].

By assuming that the initial state $|\psi_i\rangle$ of the incident field, which is characterized by the absence of dissipation ($a = 0$), is simply given by

* Corresponding author.
E-mail address: i.cacciari@ifac.cnr.it (I. Cacciari).

https://doi.org/10.1016/j.optcom.2018.01.056
Received 20 December 2017; Received in revised form 24 January 2018; Accepted 27 January 2018

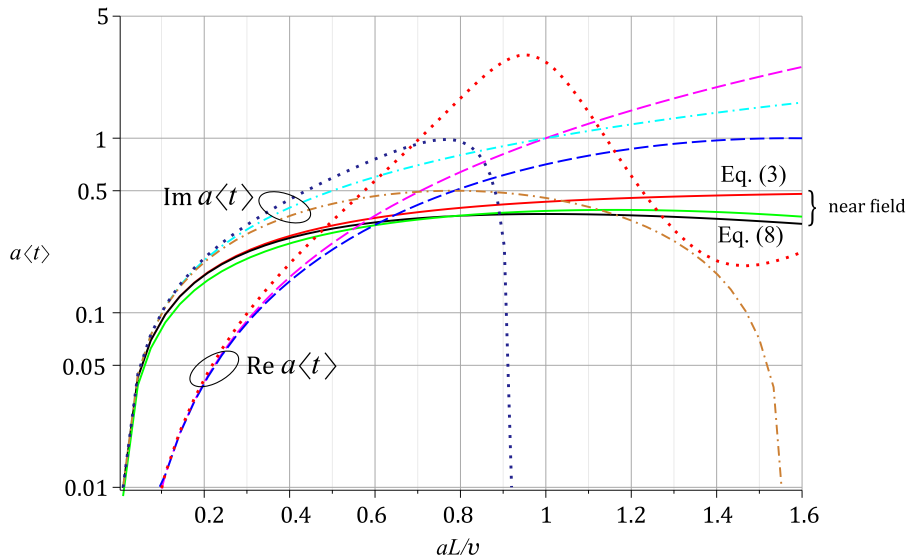

Fig. 1. Comparison of time scales relative to tunneling processes and near-field propagation. For tunneling, the real and imaginary duration of traversal time are reported as given by Eq. (4) once multiplied by a, vs. aL/v. In addition, the real and imaginary parts, as they result from an alternative treatment and as given by Eq. (5) in [7] (or Eq. (7) in [19]), are represented by dotted lines. These show a sort of resonance (with a peak in the real part and a zero in the imaginary one) near $aL/v = 1$, which finds confirmation in Refs. [7,19]. The time scale relative to propagation in the near field, as given by Eqs. (3) and (8) for $A = 0$ (black) and $A = 0.1$ (green), once multiplied by a, is represented by the relative curves. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$\delta(t-r)$, while the final state is given by $|\psi_f\rangle = g(r,t)$, the "weak value" of the observable time turns out to be [21]

$$\langle\psi_i|t|\psi_f\rangle = \int_0^{t_0}\int_0^t \delta(t-r)e^{-2ar}drdt = \frac{1}{2a}\left(1-e^{-2at_0}\right), \tag{2}$$

where we have adopted the first moment of the randomized time, as originally deduced in [11], that is, $\mu_1(t) = \int_0^t e^{-2ar}dr$, and $t_0 = L/v$ is the semiclassical traversal time. This result is exactly the same as the one already obtained in [16,18], for which the average time was given by

$$\langle t\rangle = \frac{1}{2a}(1-e^{-2aL/v}), \tag{3}$$

where L is the barrier length and v is the velocity in absence of dissipation. The "true" duration of the tunneling process has to be assumed as given by Eq. (3), once the analytical continuation $t_0 \to it_0$ ($t_0 = L/v$) has been made, namely [22]

$$\langle t\rangle = \frac{1}{2a}\left[1-\cos(2aL/v)\right] + \frac{i}{2a}\sin(2aL/v) \simeq a\left(\frac{L}{v}\right)^2 + i\frac{L}{v}, \tag{4}$$

where the last member holds for small values of the argument, i.e. for $2aL/v \ll 1$. That is, the tunneling time is found to be a complex quantity, the imaginary part of which is nearly coincident with the semiclassical traversal time, while the real part is typically a smaller quantity (second order in L/v): see Fig. 1, where the curves relative to Eqs. (3) and (4), once multiplied by a, are reported vs. aL/v. This behavior can give rise also to superluminal effects as evidenced in the experiments of Refs. [18,19], and can supply a reliable description in several cases of macroscopic quantum tunneling (Josephson junctions), as in Refs. [7,23].

3. A transition-elements analysis

A completely different approach to the problem of determining the time scales in these kinds of systems, is offered by the transition elements [14]. Starting from Eq. (7.49) in Ref. [14], which enables a visualization of typical paths for a quantum-mechanical particle, and which can be written, for $v \simeq c$, as

$$\varepsilon\langle 1\rangle = i\frac{mc^2}{\hbar}\left\langle\left(\frac{\Delta x}{v}\right)^2\right\rangle, \tag{5}$$

where $\varepsilon = \Delta t$ and $\langle 1\rangle = \exp\left(\frac{imc^2\Delta t}{\hbar}\right)$ is the propagator. By identifying $imc^2/\hbar$ with a [24], we obtain the approximate relation

$$\langle t\rangle = \Delta t(1+a\Delta t) = a\left\langle\left(\frac{\Delta x}{v}\right)^2\right\rangle, \tag{6}$$

from which we can envisage two different regimes that correspond to the two terms in the first members, i.e. $\Delta t \propto a\langle\left(\frac{\Delta x}{v}\right)^2\rangle$ which for $\Delta x = L$ tends to be coincident with the real part in Eq. (4), and the other $(\Delta t)^2 \propto \langle(\Delta x/v)^2\rangle$, i.e. $\Delta t \propto L/v$ which tends to be coincident with the absolute value of the imaginary part in Eq. (4), i.e. with the semiclassical traversal time. However, a more convincing result can be obtained here as follows.

Let us assume as that the variation in the classical action S_{cl} is due to a perturbative term that enters the transition element of the trajectory $\langle\ddot{x}\rangle$, (see Eq. (7.69) in Ref. [14]):

$$\langle\ddot{x}\rangle = \frac{\delta S'_a}{\delta f(t)}\exp\left[\frac{i}{\hbar}(S'_{cl}-S_{cl})\right]\langle 1\rangle_S, \tag{7}$$

where the symbol δ represents the functional derivative of the varied action S'_{cl} with respect to $f(t) = \eta\dot{x}(t)$, $\eta = 2ma$ and $\dot{x}(t)$ the velocity. The quantity m is the "mass of the particle". S_{cl} denotes the unperturbed action, and $\langle 1\rangle_S$ is the propagator. By retaining only the first-order terms in the development in power series of the exponential in (7), by replacing the damping parameter a with ia, and by dividing the path length L by the mean velocity v, we arrive at the following approximate result for the real part of the traversal time [15,25]:

$$\Re e\langle t\rangle \simeq \frac{L}{v}\left[1-A\cos\left(2a\frac{L}{v}\right)\right]\exp(-aL/v), \tag{8}$$

where $A \simeq a/2\omega$, ω is the angular frequency of the wave, $v \simeq c$ is the light velocity in vacuum, and $\langle 1\rangle_S \approx e^{-aL/v}$. The curves relative to Eq. (8) for $A = 0$ and $A = 0.1$, once multiplied by a, are reported vs. aL/v in the same Fig. 1 for comparison with Eqs. (3) and (4) (these also multiplied by the parameter a). It is interesting to note that the curves relative to Eq. (8) are very comparable with the one relative to Eq. (3) and that, for small values of the variable aL/v, all these curves tend to the semiclassical time multiplied by a, just aL/v, while they are decidedly different from the real part in Eq. (4) which, as said before, supplies a real duration of traversal time in tunneling. The results as