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Phase-locking dynamics in optoelectronic oscillator

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ABSTRACT

This paper analyzes the phase-locking phenomenon in single-loop optoelectronic microwave oscillators considering weak and strong radio frequency (RF) signal injection. The analyses are made in terms of the lock-range, beat frequency and the spectral components of the unlocked-driven oscillator. The influence of RF injection signal on the frequency pulling of the unlocked-driven optoelectronic oscillator (OEO) is also studied. An approximate expression for the amplitude perturbation of the oscillator is derived and the influence of amplitude perturbation on the phase-locking dynamics is studied. It is shown that the analysis clearly reveals the phase-locking phenomenon and the associated frequency pulling mechanism starting from the fast-beat state through the quasi-locked state to the locked state of the pulled OEO. It is found that the unlocked-driven OEO output signal has a very non-symmetrical sideband distribution about the carrier. The simulation results are also given in partial support to the conclusions of the analysis.

1. Introduction

Optoelectronic oscillator (OEO) provides a potential *and* convenient method of generating low phase noise RF signals [1–3], which is needed for many applications in photonic systems such as high-precision radars, wireless base stations, microwave photonic and optical signal processing. Some important applications, where OEO is used are in generation of pulsed signals and complex waveform [4–6], high-precision distance measurement [7], optical clock recovery [8], low-power RF signal detection [9], optical frequency stability measurement [10] and optical power monitoring [11]. In the last few years, lots of works have been carried out on different configurations of OEO and *so* their advantages and disadvantages are *subject of discussion*. These studies have been concentrated on the performance of the oscillator regarding phase noise [12,13], the tunability [14,15] and the physical size [16]. To reduce the phase noise of the output signal at low-offset frequencies, injection-locking technique with different configurations of OEO *has* been proposed by several researchers [12,17,18]. Injection-locking technique leads to the phase-locking phenomenon between two coupled oscillators. Recently, only Tala et al. [19] *has* investigated the phase-locking phenomenon in narrow-band OEO from the point of view of non-linear dynamics. This analytical model *cannot* describe the clear picture of phase-locking dynamics within OEO and the spectrum of the pulled OEO cannot be evaluated from *that* analysis. Deep understanding and more insight of the unlocked-driven state are required to demonstrate the locking dynamics within the OEO more accurately.

The OEO is a very important building block of modern integrated wireless communication systems such as wide-band RF trans-receiver circuits which incorporate mixers, power amplifiers and RF amplifier. The power amplifier has non-linear characteristic and its output contains a large number of frequency components in the close vicinity of the OEO carrier frequency. Thus, it is very urgent to *analyze* the phase-locking phenomenon and the associated frequency pulling effects in OEO due to RF signal injection. This knowledge will make us to use it more advantageously in intelligent potential application and simultaneously to reduce its effect whenever undesirable. Only P. Devgan et al. [20] had investigated the experimental work on the frequency pulling in OEO due to RF injection signal. As far *as* the knowledge of the author *extends*, no attempts has been made till now to quantify injection pulling and injection-locking effects in single-loop OEO. Lock-range is an important parameter for judging the merit of an injection-locked system. The lock-range can be adjusted by controlling the injected signal's amplitude. In a SONET optical network, the cut-off frequency for jitter-transfer for an OC-192 (9.953 Gbps) data signal is 120 kHz [21]. So the lock-range for an OEO in its application should be around 120 kHz, which

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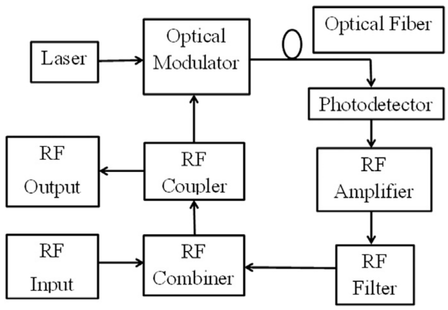

Fig. 1. Schematic diagram of a single-loop OEO.

can be obtained by injecting strong RF signal to the OEO. Thus, phase-locking mechanism should be investigated considering strong RF signal injection. In this paper, an attempt has been made to throw light on various new aspects of phase-locking dynamics and pulling effect within single-loop OEO for weak and strong injection *signals*, so far *as* not reported by other researchers.

This paper is organized as follows. In Section 2, we describe the characteristic equations of the unlocked-driven single-loop OEO to *analyze* the phase-locking phenomenon of the oscillator. The small injection analytical model of the oscillator is presented in Section 3, while in Section 4 we focus on the large injection perturbation model to *analyze* the influence of amplitude perturbation on the phase-locking dynamics. Section 5 shows the simulation results and finally, Section 6 *represents* the conclusions.

2. Characteristic equations of the unlocked-driven OEO

In this section, we derive the mathematical model of the unlocked-driven single-loop OEO and this knowledge is then utilized to describe the phase-locking and frequency pulling mechanism of the oscillator in Sections 3 and 4. As shown in Fig. 1, a single-loop OEO consists of a continuous wave (CW) semiconductor laser, a wide-band electro-optic (E/O) Mach–Zehnder modulator (MZM), a narrow-band microwave RF filter, RF amplifier, a fast photo-detector, a RF-coupler and an optical fiber *delayed* line, which is used to provide high quality (Q) factor by long-length and low-loss of the optical fiber. A stable microwave oscillation is produced *while* the overall loop-gain is higher than the losses, satisfying the Barkhausen criteria. Now, a stationary RF signal is injected in the RF combiner of the free-running OEO.

Let, the free-running frequency of the OEO be ω_O and the driving frequency be ω_1. Assume the output of the free-running OEO and the external injection signal to be, respectively, *in* the forms *of*

$$v_O(t) = V_O(t)\exp\left\{j\left(\omega_O t - \phi_O\right)\right\} \tag{1}$$

and

$$v_s(t) = V_s \exp\left\{j\left(\omega_1 t\right)\right\} \tag{2}$$

where ω_O and ϕ_O are the free-running or natural frequency and quiescent phase of the OEO, ω_1 and V_s are the frequency and fixed amplitude of the injection signal respectively. The presence of the RF injection signal modifies the operating characteristics of OEO. *This* will change the instantaneous amplitude and phase of the oscillator. The perturbed output of the OEO is taken as

$$v_O(t) = V_O(t)\exp\left\{j\left(\omega_1 t - \phi(t)\right)\right\} \tag{3}$$

where $\phi(t)$ denotes the instantaneous phase difference between the OEO and the external RF signal.

The optical power at the output of the optical modulator is

$$P(t) = 0.5\alpha P_O\left(1 - \eta \sin\left\{\pi\left[{}^{v_O(t)}/_{v_{\pi 2}} + {}^{v_S(t)}/_{v_{\pi 1}} + {}^{v_B}/_{v_{\pi 2}}\right]\right\}\right) \tag{4}$$

where P_O is the input optical power, α is the insertion loss, v_B is the dc bias voltage, *and* η is a parameter determined by the extinction ratio of the modulator $(1+\eta)/(1-\eta)$. $v_{\pi 1}$ and $v_{\pi 2}$ are modulator half-wave voltages of the driver and driven oscillators, respectively. The modulated optical signal propagates through an optical fiber *delayed* line, which induces a time delay τ corresponding to length L of fiber. The free spectral range (FSR) or mode spacing of the OEO is controlled by the length L as $FSR \cong \left(c/n_i \times L\right)$, where c is the speed of light, *and* n_i is the effective refractive index of the optical fiber. The delayed signal is detected by a fast photo-detector with a conversion factor ρ and the output electrical signal given by

$$V_{PD}(t) = \rho P(t-\tau). \tag{5}$$

We consider a first-order linear band-pass filter (BPF) having transfer function

$$G(s) = \frac{1}{1 + Q\left(\frac{\omega_O}{s} + \frac{s}{\omega_O}\right)} \tag{6}$$

to select the oscillation frequency of the OEO, where $s\,(=j\omega)$ is the complex frequency. Since the bandwidths of the BPF and RF amplifier *being* much smaller than the microwave frequency ω_O, we can neglect higher-order harmonic terms *as well as* the influence of the higher-order modes on the amplifier saturation and using (1), (2), (4) *and* (5), we obtain

$$\begin{aligned} V_{PD}(t) = {} & 2\eta V_{ph}\sin\left(\frac{\pi v_B}{V_{\pi 2}}\right)\Big[J_O\left(V_{s\pi}\right)J_{C_1}\left(V_{O\pi}(t)\right)\exp\left(-j\Omega\tau\right) \\ & + J_1\left(V_{s\pi}\right)J_{C_O}\left(V_{O\pi}(t)\right)\exp\left\{-j\left(\Omega\tau+\phi\right)\right\}\Big]\, v_O(t). \end{aligned} \tag{7}$$

Hence the parameters *belonging to* (7) are the photo-detector voltage $V_{ph}\left(=\frac{1}{2}\alpha P_O\rho\right)$, normalized injection signal amplitude $V_{s\pi} = \left(\pi V_s/V_{\pi 1}\right)$, normalized output signal amplitude $V_{O\pi} = \left(\pi V_O/V_{\pi 2}\right)$, $\Omega = \left(\omega_1 - \omega_O\right)$ is the frequency detuning between the free-running OEO and the injection signal. $J_{C_O}(x) = \frac{J_O(x)}{x}$ and $J_{C_1}(x) = \frac{J_1(x)}{x}$ are the zero-order and first-order Bessel Cardinal functions respectively, where $J_O(x)$ and $J_1(x)$ are the zero-order and first-order Bessel function of first kind respectively. The relation between the output electrical signal of the RF amplifier with gain G_A and the output signal of the photo-detector can be written as

$$V_{PD}(t) = \frac{v_O(t)}{G_A G(s)}. \tag{8}$$

Using (3), (6), (7) into (8) and equating the real and imaginary parts, we have the following two equations

$$\frac{da(t)}{dt} = \mu\left[\left(\gamma - (\gamma-1)a^2(t)\right)\cos\Omega\tau - 1\right]a(t) + d\cos\left(\Omega\tau + \phi(t)\right) \tag{9}$$

$$\frac{d\phi(t)}{dt} = \Omega + \mu\left[\left(\gamma - (\gamma-1)a^2(t)\right)\right]\sin\Omega\tau - \frac{d}{a(t)}\sin\left(\Omega\tau + \phi(t)\right) \tag{10}$$

where $\gamma = \frac{\eta\pi V_{ph}G_A}{V_{\pi 2}}\sin\left(\frac{\pi v_B}{V_{\pi 2}}\right)$ is the effective loop-gain, $d = \mu\gamma V_{s\pi}/V_{Os\pi}$, $a(t) = V_{O\pi}(t)/V_{Os\pi}$ and $V_{Os\pi} = 2\sqrt{2\left(1-\frac{1}{\gamma}\right)}$. From (9) and (10), it is clear that the injection RF signal perturbs both the amplitude and phase of the OEO. Here, (9) and (10) are two coupled non-linear differential equations and their solutions cannot be obtained considering both amplitude and phase perturbation simultaneously. In the absence of injection signal, the OEO oscillates with the unperturbed normalized steady amplitude $V_{Os\pi}$, which can be obtained by putting $d = 0$ and $\frac{da(t)}{dt} = 0$ in Eq. (9).

3. Small injection analytical model

In this section, we present the small injection perturbation analytical model of unlocked-driven OEO. *This* analytical model is then used to describe the frequency pulling phenomenon of the driven oscillator. To begin with, let us assume *both* that (1) the strength of the injection signal