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# A calibration method for fringe reflection technique based on the analytical phase–slope description

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ABSTRACT

The fringe reflection technique (FRT) has been one of the most popular methods to measure the shape of specular surface these years. The existing system calibration methods of FRT usually contain two parts, which are camera calibration and geometric calibration. In geometric calibration, the liquid crystal display (LCD) screen position calibration is one of the most difficult steps among all the calibration procedures, and its accuracy is affected by the factors such as the imaging aberration, the plane mirror flatness, and LCD screen pixel size accuracy. In this paper, based on the deduction of FRT analytical phase–slope description, we present a novel calibration method with no requirement to calibrate the position of LCD screen. On the other hand, the system can be arbitrarily arranged, and the imaging system can either be telecentric or non-telecentric. In our experiment of measuring the 5000mm radius sphere mirror, the proposed calibration method achieves 2.5 times smaller measurement error than the geometric calibration method. In the wafer surface measuring experiment, the measurement result with the proposed calibration method is closer to the interferometer result than the geometric calibration method.



## 1. Introduction

Characterizing surface appearance is critical in nowadays manufacture industries [1–4]. The fringe reflection technique (FRT), or known as phase measuring deflectometry, has been one of the most popular specular surface measurement methods [5–8]. It has the advantages of being non-contact, full field measurement and high dynamic range. In the FRT system, a liquid crystal display (LCD) screen is usually employed to display a computer-generated fringe pattern. A digital camera is then utilized to capture the fringe pattern reflected by the test specular surface. With the well-calibrated system parameters, the test surface slopes can be calculated according to the phase of the captured fringe pattern.

The calibration accuracy has been a critical factor to restrict the overall measurement accuracy of FRT. The existing calibration methods in FRT [9–12] usually contain two parts, which are camera calibration and geometric calibration. The camera intrinsic parameters including focal length, principle point location, pixel skew factor and pixel size are calculated by camera calibration. The geometric calibration in FRT is to locate the position of reference plane mirror and the LCD screen in camera coordinate system. The LCD screen position calibration is the hardest step within the whole calibration procedures. A popular way to calibrate LCD screen position can be described as following steps. A calibration plate pattern with a series of known distance circles is displayed on the LCD screen. A well-calibrated camera captures this calibration plate pattern reflected by a plane mirror. The rotation and translation matrixes of the virtual LCD screen plane (caused by the mirror reflection) under the camera coordinate system can be obtained accordingly. Then, the virtual LCD screen plane is required to be transferred to the real LCD screen plane. The accuracy of rotation and translation matrixes is related to the camera calibration accuracy, the imaging aberration, the plane mirror flatness, and accuracy of the distance between two neighboring circles. The distance between two circles equals the number of pixels between two circles times the size of screen pixel. However, due to the inflexibility of calibration the screen pixel size, the nominal value of pixel size given by the screen producer is usually used directly. Therefore, the calibration of the LCD screen position is inflexible and hard to achieve high accuracy.

In this paper, based on the FRT phase–slope analytic description, a novel calibration method is proposed. In the proposed calibration method, there is no requirement to know the position of LCD screen.

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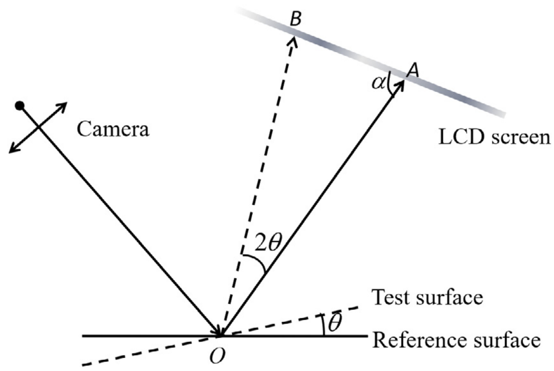


**Fig. 1.** Schematic of FRT system and the principle of calculating the surface slope.

Section 2 of this paper shows the deduction of the FRT phase–slope analytic description. In Section 3, the proposed calibration method is described. The experiment is given in Section 4.

## 2. Phase–slope relation of FRT

In FRT, the camera records the fringe pattern on the LCD screen via a specular surface. Due to the reversibility of optical path, we can suppose a ray of light 'emits' from a camera pixel. After reflected by the reference surface at point $O$, it reaches the LCD screen at position $A$, as shown in Fig. 1. If there is an angle difference $\theta$ between the test surface and reference surface at position $O$, the light will deflect an angle of $2\theta$ and will reach the LCD screen at point $B$.

In $\Delta OAB$, there is

$$|AB| = \sin 2\theta \frac{|OA|}{\sin(\pi - \alpha - 2\theta)}. \tag{1}$$

The phase difference between $A$ and $B$ is

$$\Delta\varphi = |AB| \frac{2\pi}{P}, \tag{2}$$

where $P$ donates the period of the fringe pattern on LCD screen. With Eqs. (1) and (2), we have

$$\Delta\varphi = \frac{2\pi\,|OA| \sin 2\theta}{P \sin(\pi - \alpha - 2\theta)} = \frac{2\pi\,|OA| \tan 2\theta}{P(\sin\alpha + \tan 2\theta \cos\alpha)}, \tag{3}$$

which can be written as

$$\tan 2\theta = \frac{P \sin\alpha \Delta\varphi}{2\pi\,|OA| - P\cos\alpha\Delta\varphi}. \tag{4}$$

In order to reconstruct the surface height from the slope, two perpendicular slope distributions are usually required. Therefore, the LCD screen usually displays fringe patterns in two perpendicular directions. In the two directions $x$ and $y$,

$$\tan 2\theta_x = \frac{P_x \sin\alpha_x \Delta\varphi_x}{2\pi\,|OA| - P_x \cos\alpha_x \Delta\varphi_x}. \tag{5}$$

$$\tan 2\theta_y = \frac{P_y \sin\alpha_y \Delta\varphi_y}{2\pi\,|OA| - P_y \cos\alpha_y \Delta\varphi_y}. \tag{6}$$

Eqs. (5) and (6) are the phase–slope model for the arbitrarily arranged system. They show a clear relation between slope, phase difference and the related system parameters. The directions $x$ and $y$ are chosen as the directions of $x$-axis and $y$-axis of camera coordinate system.

It should be noted that Eq. (4) has not taken phase ambiguous problem [13] (The phase value will not only change with the surface slope, but also change with surface height variation) into account. While deducing Eq. (4), we have assumed that $|OA|$ is much larger than the test surface height $h$, so the surface height variation has not been considered.

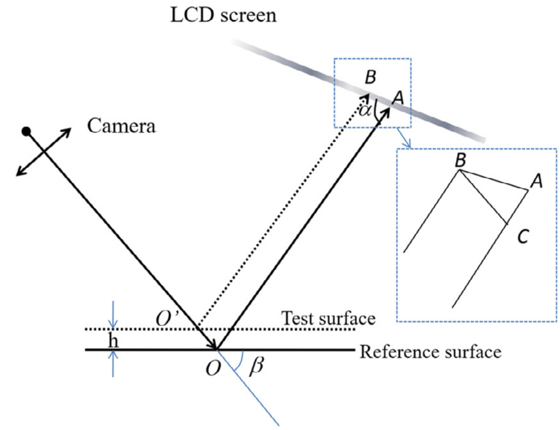


**Fig. 2.** Schematic of FRT system when there is only height variation.

In point $O$ of Fig. 1, we think there is only angle difference $\theta$ between reference surface and test surface, but there is no height difference between them. So, Eq. (4) is actually only valid for measuring quasi-plane surfaces.

Now suppose the test surface only has height difference against the reference surface, and there is no slope difference between them. As shown in Fig. 2, a ray of light 'emits' from a camera pixel. After reflecting by the reference surface at point $O$, it reaches the LCD screen at position $A$. When measuring the test surface, the light will be reflected at point $O'$, and reaches the LCD screen at position $B$. In $\Delta ABC$, the point $C$ is chosen to satisfy $BC//OO'$, and $\angle BCA = \pi - 2\beta$. So,

$$|AB| = \sin\angle BCA \frac{|BC|}{\sin\alpha} = \frac{2h\cos\beta}{\sin\alpha}. \tag{7}$$

The phase difference between $A$ and $B$ is

$$\Delta\varphi = \frac{4\pi h \cos\beta}{P \sin\alpha}. \tag{8}$$

Now think the test surface has angle difference $\theta$ against reference surface at point $O'$ in Fig. 2. However, the phase difference cannot be simply represented by Eq. (3) plus Eq. (8), because the $|OA|$ in Eq. (3) has changed to $|O'B|$ (in Fig. 2). $|OA|$ and $|O'B|$ have the relation of Eq. (9).

$$|O'B| = |OA| - |AC| = |OA| - \frac{h\sin(2\beta - \alpha)}{\sin\alpha \sin\beta}. \tag{9}$$

The total phase difference is

$$\begin{aligned} \Delta\varphi &= \frac{4\pi h\cos\beta}{P\sin\alpha} + \frac{2\pi\,|O'B| \tan 2\theta}{P(\sin\alpha + \tan 2\theta\cos\alpha)} \\ &= \frac{2\pi}{P}\left\{\frac{2h\cos\beta}{\sin\alpha} + \frac{[|OA| - \frac{h\sin(2\beta-\alpha)}{\sin\alpha\sin\beta}]\tan 2\theta}{(\sin\alpha + \tan 2\theta\cos\alpha)}\right\}. \end{aligned} \tag{10}$$

Eq. (10) shows that both slope and height will affect the phase difference. We can analyze how surface height variation $h$ affects the phase difference by Eq. (10). In order to acquire the unwrapped phase from the recorded fringe patterns, a reliability-guided phase unwrapping method [14] is employed in this paper, and if the test surface has discontinuity area, the dual-frequency or multi-frequency phase unwrapping method [15,16] should be taken.

## 3. A calibration method based on the phase–slope analytic description

As discussed in Section 2, Eq. (4) can have high accuracy for measuring quasi-plane surface. Considering the system is arbitrarily arranged, and the imaging system could be non-telecentric, for each